Course Overview

This course examines the fundamentals and basic physical operation of electrical transformers used in the electric power systems. The ancillary concerns of single-phase, auto- and three-phase transformers, needed for the practical application of all power system components are also discussed. Specialized instrument transformers including current and potential transformers are investigated. The harmonics of in the power transformers and inrush currents are also discussed. The main constructions and design procedure of the transformer are introduced in this course.

This course is intended for third year undergraduate students who have a previous course in energy conversions and electrical machines such as EPM2106 and EPM2208 or work experience in fundamental of electric power system engineering such as EPM2105 and EPM2207.

Administrative Information

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References:

[R1]	Principles of Electric Machines and Power Electronics, 3rd Edition by P. C. Sen, John Wiley and Sons, 2012, ISBN: 978-1-118-07887-7
[R2]	Electric Machinery and Transformers, 3rd Edition by B. S. Guru and Husayin R. Hiziroglu, Oxford University Press, 2001, ISBN: 978-0-19-513890-0
[R3]	Electric Machinery Fundamentals, 5th Edition by Stephen J. Chapman, McGraw-Hill, 2012, ISBN: 978-0-07-352954-7
[R4]	Electrical Machines Theory and Practice, 1st Edition by M. N. Bandyopadhyay, PHI learning, New Delhi-110001, 2009
[R5]	Electrical Technology Handbook, "Parallel Operation of Single-phase Transformers", Thuraja
[R6]	Lectures on Electric Machines-1, Dr. Aly Eltamaly, KSU, KSA
[R7]	المرجع فى محولات القدرة الكهربية ، أ.د. محمود جيلانى ، جامعة القاهرة

Grading and Exam Schedule

Grade Components:

Assignments and Activities (approximately weekly): 10% (20 Points)

Hour Exams: 10% (20 Points)

Oral and laboratory Exam: 20% (40 Points)

Final Exam: 60% (120 Points)

Hour Exams Schedule:

Hour Exam #1 Lecture-1, Week 4

Hour Exam #2 Week-7

Hour Exam #3 Lecture-1, Week-11

Course Outline

Week #	Date	Lecture Topic
1	Lec. 1	Principles of Transformers: Introduction, Basic Construction of a Transformer, Types of Transformers
	Lec. 2	
2	Lec. 1	The Ideal Transformer: EMF Equation, Turns Ratio, Effect of Load Current, Active and Reactive Power, Impedance Transfer
	Lec. 2	Analysis of Circuits Containing Ideal Transformer, Transformer Polarity, Transformer Ratings
3	Lec. 1	Practical or Real Transformers: Theory of operation, Transformer Equivalent Circuits (1)
	Lec. 2	Transformer Equivalent Circuits (2)
4	Lec. 1	*Hour Exam #1*
	Lec. 2	Determination of Transformer Equivalent Circuit Parameters
5	Lec. 1	Efficiency, Maximum Efficiency, All-Day Efficiency
	Lec. 2	Voltage Regulation, Transformer Taps and Voltage Regulator
6	Lec. 1	Per-Unit System
	Lec. 2	Parallel Operations of Single-Phase Transformers
7		*Hour Exam #2 (Mid-Term Exam Week)*
8	Lec. 1	Auto-transformers: Introduction, Voltage and Current Relationships, Apparent Power Ratings, Saving of Copper, Advantages and Disadvantages
	Lec. 2	Three-Phase Transformers: Introduction, Transformer Winding Connection Designations, Connections, Vector Grouping.
9	Lec. 1	Analysis of Three-Phase Transformer using a Single-

		Phase Equivalent-Circuit (1).
	Lec. 2	Analysis of Three-Phase Transformer using a Single-Phase Equivalent-Circuit (2).
10	Lec. 1	Harmonics and Transient Inrush Current in Transformers.
	Lec. 2	Instrument Transformers
11	Lec. 1	*Hour Exam #3*
	Lec. 2	Selected Topics in Transformer Engineering
12	Lec. 1	Design of Electrical Transformers
	Lec. 2	
13	Lec. 1	
	Lec. 2	

Chapter 1

Ideal Transformer

Chapter 1

Principles of Transformers

1.1 Introduction

The transformer is a static device that transforms the AC electrical power at one frequency and voltage level to an AC electric power at the same frequency and another voltage level through the action of a magnetic field.

The transformer essentially consists of two windings wrapped around a common ferromagnetic core and coupled by a mutual magnetic field as shown in Fig. 1.1. One of these windings is called primary winding and the other is called secondary winding. The primary is the winding, which receives electric power, and the secondary is the one, which may deliver it.

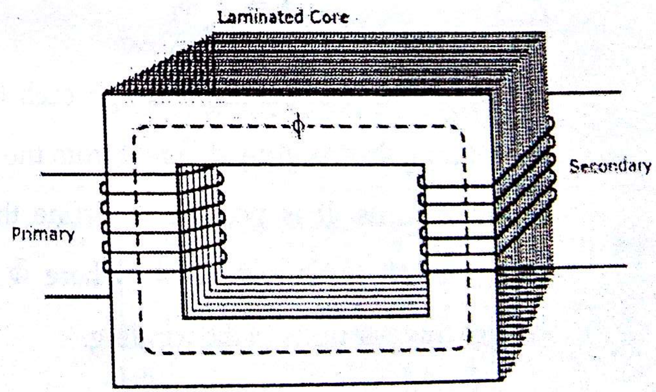


Fig. 1.1. Diagram showing magnetic circuit and windings of a transformer

The transformer principle is based on the work of Michael Faraday (1791-1867), whose discoveries in electromagnetic induction showed that, given two magnetically coupled coils, a changing current in one coil would induce an electromagnetic force in the other coil.

If the primary winding is connected to an AC voltage source, an alternating flux (Φ) is setup in the core, most of which is linked up with the secondary winding in which it produces mutually induced *emf* (e) according to Faraday's laws as follows:

$$e = \frac{d\lambda}{dt}$$

where, e is the induced emf, and

λ is the flux linkage in the winding across which the voltage is being induced. The flux linkage λ is the sum of the flux passing through each turn in the windings added over all the turns of the winding:

$$\lambda = \sum_{l=1}^{N} \Phi_l$$

Although, the flux passing through each turn of the winding Φ_i is slightly different from the flux in the other turns. It is possible to define the flux linkage in all turns as $\lambda = N\,\bar{\Phi}$, where $\bar{\Phi}$ is the average flux per turns in the winding.

If the secondary circuit of the transformer is closed, a current flow in it and so electric energy is transferred from the primary winding to the secondary winding.

In short, a transformer carries the operations shown below:

1. *Transfer of electric power from one circuit to another.*
2. *Transfer of electric power without any change in frequency.*
3. *Transfer with the principle of electromagnetic induction.*
4. *The two electrical circuits are linked by mutual induction.*

1.2 Basic Construction of a Transformer

The basic construction of a transformer consists of **two windings** having *mutual inductance* and a **laminated steel core**. The two windings are insulated from each other and from the core. The transformer will also need some **suitable container** for the assembled core and windings, an **insulating medium** with which the core and its windings from its container can be insulated. *In order to insulate and to bring out the terminals of the winding from the tank,* **bushings** that are made from either porcelain or capacitor type must be used [see Fig. 1.2].

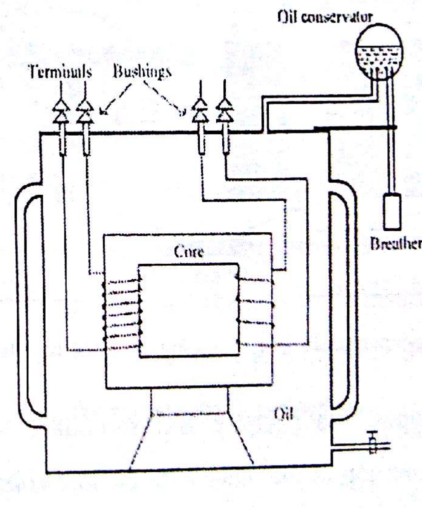


Fig. 1.2. Basic construction of the transformer

1.3 Types of Transformers

Transformers can be classified on different basis, like types of core construction, purpose, types of supply, types of cooling etc.

1.3.1 On the basis of construction

The transformers can be classified according to its core construction into two types as; (i) core-type and (ii) shell-type transformers, which are described below.

The *core-type transformer* consists of a simple rectangular laminated core with cylindrical windings wrapped around the two sides of the rectangle as shown in Fig. 1.3. On the other hand, *the shell-type transformer* consists of a three-legged laminated core with the windings wrapped around the center leg as shown in Fig. 1.4.

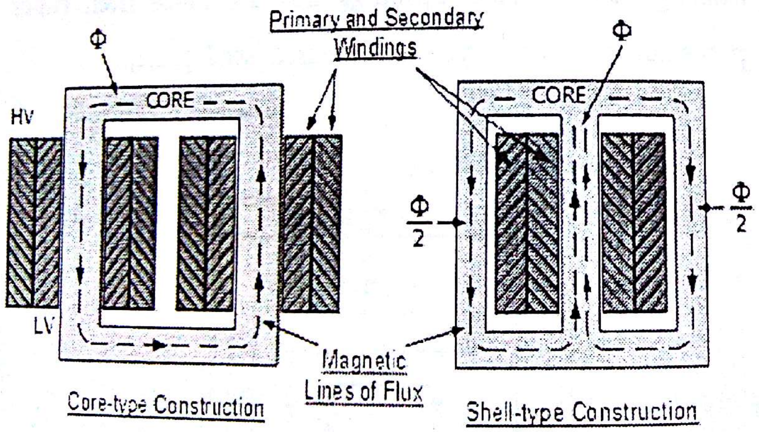


Fig. 1.3. Core-type transformer   Fig. 1.4. Shell-type transformer

In both types, the primary and secondary windings are wrapped one on top of the other with the low voltage windings

are placed nearer to the core as shown in Figs. 1.3 and 1.4. Such an arrangement serves the following two purposes:

1. It simplifies the problem of insulating the high voltage winding from the core. In other words, the low voltage winding is the easiest to insulate.
2. It reduces the leakage flux.

> *The selection between the core and shell type is made by comparing the cost because similar characteristics can be obtained from both types.*

Most manufacturers prefer to use shell-type transformers for high-voltage applications or for multi-winding design.

1.3.2 On the basis of their purpose

1. Step-up transformer: Voltage increases at secondary.
2. Step-down transformer: Voltage decreases at secondary.

1.3.3 On the basis of type of supply

1. Single phase transformer
2. Three phase transformer

1.3.4 On the basis of their use

1. Power transformer: Used within the generating station to step-up the generated voltage at the transmission voltage level, it can be also called a unit transformer. It has been used also in the transmission networks.

2. Distribution transformer: Used in distribution network, comparatively lower rating than that of power transformers.
3. Instrument transformer: Used in relay and protection purpose in different instruments in industries
   - Current transformer (CT)
   - Potential transformer (PT)

1.3.5 On the basis of cooling employed

Transformers can also be classified according to the type of cooling employed. The different types according to these classifications are:

1. Oil-filled self-cooled type
2. Oil-filled water cooled type
3. Air blast type (air cooled)

1.4 The Ideal Transformer

The ideal transformer is a transformer, which has no losses and no leakage flux. In other words, the ideal transformer has the following properties:

1. The winding resistances are negligible. Therefore, *the copper losses are negligible.*
2. All fluxes are confined to the core and link both windings; that is, no leakage fluxes are present. Therefore, *the core losses are negligible.*
3. The permeability of the core is infinite (i.e., $\mu \to \infty$). Therefore, the exciting current required to establish flux in the core is negligible. This means, *the net mmf required to establish a flux in the core is zero.*
4. The magnetization curve must have the shape shown in Fig. 1.5. Notice that for an unsaturated core the net *mmf* force $mmf_{net} = 0$, implying that $N_1 i_1 = N_2 i_2$.

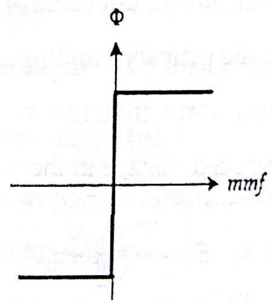


Fig. 1.5. The magnetization curve of an ideal transformer

It may be noted that it is impossible to utilize an ideal transformer in practice.

1.4.1 EMF Equation of an Ideal Transformer

When the primary winding is connected to an AC supply with a sinusoidal voltage v_1, while the secondary winding is <u>left open</u>, a time-varying flux Φ is established in the core. Assume that, the core flux Φ varies sinusoidally with time. Thus,

$$\Phi(t) = \Phi_m \sin \omega t$$

where Φ_m is the amplitude of the core flux
$\omega = 2\pi f$ is the angular frequency
f is the frequency

According to Faraday's law, a voltage e_1 will be induced in the primary winding as follows:

$$e_1(t) = N_1 \frac{d}{dt}\Phi(t)$$

$$= N_1 \omega\, \Phi_m \cos \omega t$$

$$= E_{1m} \cos \omega t$$

where N_1 is the number of turns of the primary winding
E_{1m} is the maximum value of the induced voltage in the primary winding

Therefore, the phasor form in terms of its root-mean-square (rms) value of the induced voltage in the primary is

$$E_1 = \frac{N_1 \Phi_m \omega}{\sqrt{2}} \angle 0^\circ$$

$$= 4.44 N_1 f \Phi_m \angle 0^\circ$$

The core flux also links the secondary winding and induces an AC voltage e_2

$$e_2(t) = N_2 \frac{d}{dt}\Phi(t)$$

Similarly, the rms value of the emf induced in the secondary is

$$E_2 = 4.44 N_2 f \Phi_m \angle 0^\circ$$

In an ideal transformer

$$V_1 = E_1$$

$$V_2 = E_2$$

where V_1 and V_2 are the rms values of the primary and secondary terminal voltages.

In an ideal transformer, the terminal voltages are in-phase as shown in the phasor diagram of Fig. 1.6.

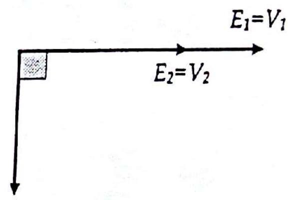


Fig. 1.6. Phasor diagram of ideal transformer at no-load

1.4.2 Turns Ratio (a)

From the previous section, the idealized case is assumed and the induced emfs in the primary and secondary are equal to the corresponding terminal voltage. Thus, the turns ratio or the voltage transformation ratio is

$$\frac{v_1}{v_2} = \frac{e_1}{e_2} = \frac{N_1}{N_2} = a$$

In terms of phasor quantities, this relation is

$$\frac{V_1}{V_2} = \frac{E_1}{E_2} = \frac{N_1}{N_2} = a$$

(a) If $N_2 > N_1$, i.e., $a < 1$, the transformer is a step-up

(b) If $N_2 < N_1$, i.e., $a > 1$, the transformer is a step-down

(c) If $N_1 = N_2$, i.e., $a = 1$, the transformer is matching

It can be noted that the voltages in the windings of an ideal transformer are directly proportional to the turns of the windings.

EXAMPLE 1.1

A single-phase, 4600/460 V, 60 Hz, ideal transformer is connected to a 1ϕ, 60 Hz, 4600 V power supply. The maximum flux density in the core is 0.85 T. If the induced voltage per turn in 10 V, determine:

(a) The primary turns (N_1) and the secondary turns (N_2).

(b) The cross-sectional area (A_c) of the core.

Solution

(a) Since

$$V_1 = E_1 = 4600\ V$$

$$V_2 = E_2 = 460\ V$$

and

$$\frac{V_1}{N_1} = \frac{V_2}{N_2} = 10$$

Therefore

$$N_1 = \frac{4600}{10} = 460$$

$$N_2 = \frac{460}{10} = 46$$

(b) Since

$$B_m = \frac{\Phi_m}{A_c} = 0.85 \xrightarrow{yields} A_c = \frac{\Phi_m}{0.85}$$

and

$$V_1 = E_1 = 4.44 N_1 f \Phi_m$$

$$A_c = V_1/(4.44 N_1 f B_m)$$

$$A_c = 4600/(4.44 \times 460 \times 60 \times 0.85) = 0.0442\ m^2$$

1.4.3 Effect of Load Current on Ideal Transformer

If the transformer is connected to a load as shown in Fig. 1.7, a current i_2 will flow in the secondary winding. The magnitude of i_2 depends upon the load impedance. However, its direction is such that it tends to weaken the core flux Φ and decreases the induced emf in the primary e_1. However, in the ideal transformer, *e_1 must always be equal to v_1*. In other words, *the flux in the core must always be equal to its original no-load value*. In order to restore the flux in the core to its original no-load value, the source v_1 forces a current i_1 in the primary winding, as indicated in the Fig. 1.7.

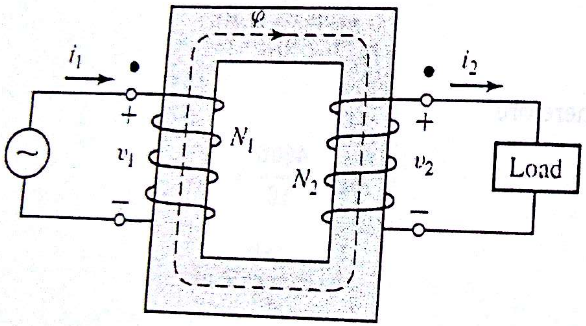


Fig. 1.7. An idealized transformer under load

In accordance to our assumptions, the mmf of the primary current mmf_1 must be equal and opposite to the mmf of the secondary mmf_2. That means,

$$N_1 i_1 = N_2 i_2$$

$$\frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{1}{a}$$

In terms of phasor quantities, this relation is

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{a}$$

The phasor diagram of an ideal transformer for current i_2 lagging lags the terminal voltage v_2 by θ_2 is shown in Fig. 1.8.

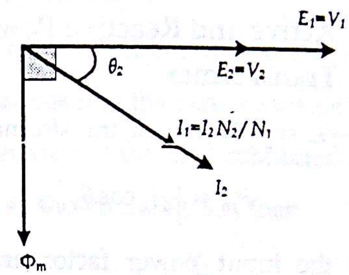


Fig. 1.8. Phasor diagram of ideal transformer at loading conditions

Note that:

The currents in the windings of an ideal transformer are inversely proportional to the turns of the windings.

Also, note that, the instantaneous power input to the ideal transformer equals the instantaneous power output from it. This means:

$$v_1 i_1 = v_2 i_2$$

In terms of phasor quantities, this relation is

$$V_1 I_1 = V_2 I_2$$

This is expected, because all power losses are neglected in an ideal transformer.

Note that:

Although there is no physical connection between the load and the supply, as soon as the load consumes power, the same power is drawn from the supply. The transformer, therefore, provides a physical isolation between the load and supply while maintaining electrical continuity.

1.4.4 Active and Reactive Power in an Ideal Transformer

The active power supplied to the transformer P_{in} is given by

$$P_{in} = V_1 I_1 \cos\theta_1$$

where $\cos\theta_1$ is the input power factor and θ_1 is the angle between the primary voltage and current. The active power supplied to the load by the transformer P_{out} is given by

$$P_{out} = V_2 I_2 \cos\theta_2$$

where $\cos\theta_2$ is the load power factor and θ_2 is the angle between the secondary voltage and current.

Since in the ideal transformer, the primary and secondary voltages and currents are in-phase respectively. Therefore, *the primary and secondary windings of the transformer have the same p.f.*

$$\frac{P_{out}}{P_{in}} = \frac{V_2 I_2 \cos\theta_2}{V_1 I_1 \cos\theta_1}$$

$$P_{out} = P_{in}$$

Thus, the output active-power of an ideal transformer is equal to its input active-power. Hence, the efficiency of the ideal transformer is 100%.

Similarly, the same relationship applies to reactive power Q and the apparent power S as follows:

$$Q_{out} = V_2 I_2 \sin\theta_2 = V_1 I_1 \sin\theta_1 = Q_{in}$$

$$S_{out} = V_2 I_2 = V_1 I_1 = S_{in}$$

1.4.5 Impedance Transfer through a Transformer

The impedance of an AC circuit can be defined as the ratio of the phase voltage across it to the phasor current flowing through it. If Z_2 is the impedance of the load connected to the secondary of the transformer as shown in Fig. 1.9, then

$$Z_2 = \frac{V_2}{I_2}$$

While, the impedance seen by the AC source is

$$Z_{in} = \frac{V_1}{I_1}$$

Since the primary voltage and current can be expressed as

$$V_1 = aV_2$$

$$I_1 = \frac{I_2}{a}$$

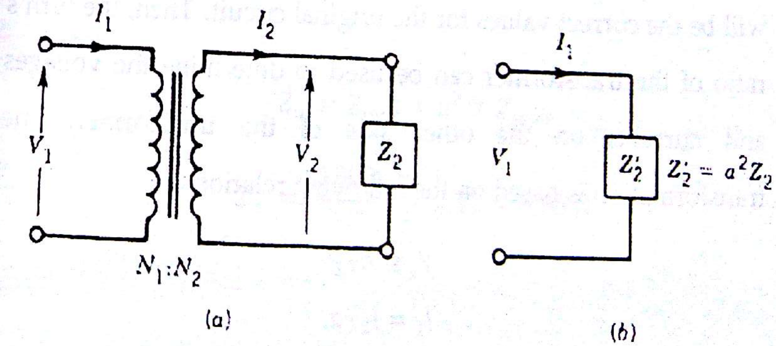


Fig. 1.9. Impedance transfer across an ideal transformer

These yields

$$Z_{in} = \frac{aV_2}{I_2/a} = a^2\frac{V_2}{I_2}$$

$$\boxed{Z_{in} = Z_2' = a^2 Z_2}$$

This equation states that the load impedance as seen by the source on the primary side is equal to a^2 times the actual load impedance on the secondary side. This equation also states that a transformer can also be used for impedance matching. A known impedance can be raised or lowered to match the rest of the circuit for maximum power transfer.

1.4.6 Analysis of Circuits Containing Ideal Transformer

In order to analyze the circuits containing an ideal transformer for its voltages and currents, an equivalent circuit with the same terminal characteristics replaces the part of the circuit on one side of the ideal transformer. After that, the new circuit (without a transformer present) can be analyzed. In the part of the circuit that was not replaced, the solutions obtained will be the correct values for the original circuit. Then, the turn's ratio of the transformer can be used to determine the voltages and currents on the other side of the transformer. The transformation is based on the following relations

$$V_1 = aV_2$$

$$I_1 = I_2/a$$

$$Z_{in} = a^2 Z_2$$

EXAMPLE 1.2

A single-phase, two-winding transformer has 1000 turns on the primary and 500 turns on the secondary. The primary winding is connected to a 220V supply and the secondary winding is connected to a 5kVA load. The transformer can be considered ideal.

a) Determine the load voltage.

b) Determine the load impedance.

c) Determine the input impedance seen by the supply.

Solution

(a) $N_1 = 1000, N_2 = 500, V_1 = 220V$ and $S_{load} = 5000$

$$V_2 = \frac{N_2}{N_1}V_1 = \frac{500}{1000} \times 220 = 110\,V$$

(b)

$$I_2 = \frac{S_{load}}{V_2} = \frac{5000}{110} = 45.45\,A$$

$$Z_{load} = \frac{V_2}{I_2} = \frac{110}{45.45} = 2.42\,\Omega$$

(c)

$$Z_{in} = Z'_{load} = a^2 \times Z_{load}$$

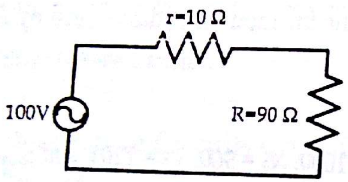
$$Z_{in} = \left(\frac{1000}{50}\right)^2 \times 2.42 = 9.68\,\Omega$$

EXAMPLE 1.3

A resistive load of 90 Ω is connected to an AC supply of 100 V with internal resistive impedance of 10 Ω. (a) Determine the power absorbed by the load. (b) To maximize the power transfer to the load, a transformer of 1:3 turns ratio is used between source and load. Determine the new power taken by the load.

Solution

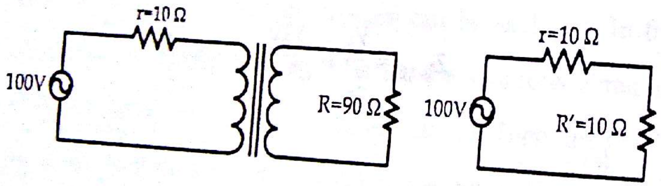
(a) For case-1



$$I = \frac{V}{R+r} = \frac{100}{90+10} = 1\,A$$

$$P = I^2R = (1)^2 \times 90 = 90\ W$$

(b) If the load resistance is connected the supply through the transformer



The load resistance referred to the primary side is

$$R' = (1/3)^2 \times 90 = 10\ \Omega$$

Therefore

$$I = V/(R'+r) = 100/(10+10) = 5\ A$$

$$P = I^2R' = (5)^2 \times 10 = 250\ W$$

1.5 Transformer Polarity

The electrical transformers may have multiple windings that *may be connected either in series to increase the voltage rating* or *in parallel to increase the current rating*. Before the connections are made, it is necessary that the polarity of each winding is determined.

1.5.1 Definition of Transformer Polarities

For the two winding transformer shown in Fig. 1.10, terminals 1 and 3 are identical, because the currents entering these terminals produce fluxes in the same direction in the core that forms the common magnetic path. Similarly, terminals 2 and 4 are identical. If these two windings are linked by a common time-varying flux, the induced voltages in these windings e_{12} and e_{34} are in phase.

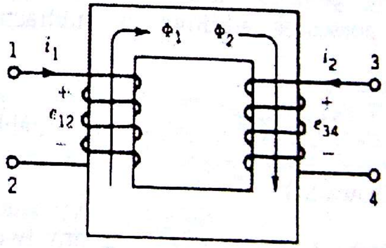


Fig. 1.10. Polarity determination

Identical terminals such as 1 and 3 or 2 and 4 are sometimes marked by dots or ± as shown in Fig. 1.11. These are called the *polarity markings* of the windings. They indicate how the windings are wound on the core. (Note that, the polarity marks are defined also with the terminals (X_1, X_2 *and* H_1, H_2)).

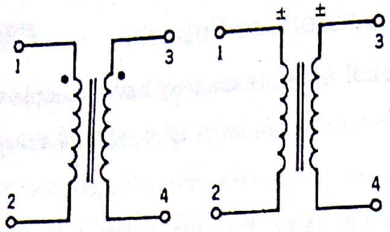


Fig. 1.11. Polarity markings of the windings

1.5.2 Determination of Polarities Experimentally

If the windings can be visually seen in a transformer, the polarities can be determined. However, usually only the terminals of the windings are brought outside the machine. Nevertheless, it is possible to determine the polarities of the windings experimentally using two tests.

Figure 1.12 shows the connections for the first experiment to determine the transformer polarities. To determine whether a transformer possesses additive or subtractive polarity, we proceed as follows:

1. Connect the *HV* winding to a suitable voltage from an AC source, V_1.
2. Connect a jumper *J* between any two adjacent *HV* and *LV* terminals.
3. Connect a voltmeter E_X between the other two adjacent *HV* and *LV* terminals.
4. Connect another voltmeter E_P across the *HV* winding. If E_X gives a higher reading than E_P, the polarity is additive. This tells us that H_1 and X_1 are diagonally

opposite. On the other hand, if E_X gives a lower reading than E_P, the polarity is subtractive, and terminals H_1 and X_1 are adjacent.

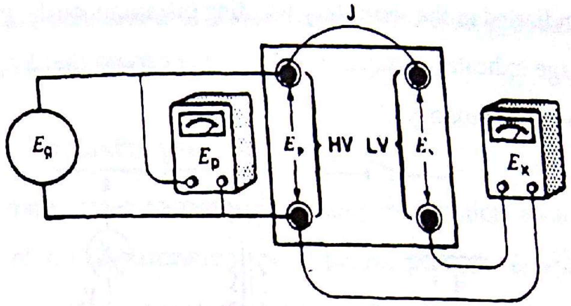


Fig. 1.12. Determining the polarity of a transformer using an ac source.

In this polarity text, jumper J effectively connects the secondary voltage E_S in series with the primary voltage E_P. Consequently, ES either adds to or subtracts from E_p. In other words, $E_X = E_P + E_S$ or $E_X = E_P - E_S$, depending on the polarity. We can now see how the terms additive and subtractive originated.

In making the polarity test, an ordinary 120 V, 60 Hz source can be connected to the *HV* winding, even though its nominal voltage may be several hundred kilovolts.

Exercise

During a polarity test on a 500 *kVA*, 69 *kV*/600 *V transformer (Fig. 1.12), the following readings were obtained:* E_P = 118 *V*, E_X = 119 *V*. *Determine the polarity markings of the terminals.*

There is another simple polarity test, which is described by Fig. 1.13. The DC voltage is applied to the primary winding of the transformer. Now, due to the transient effect, a voltage will be indicated in the secondary winding instantaneously. If the DC voltage indicates voltage, it is then quite certain that 1 and 3 are of the same polarity.

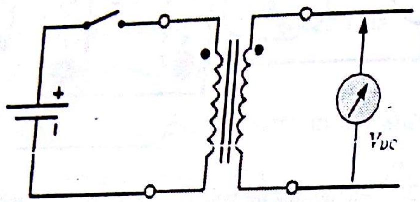


Fig. 1.13. The polarity determination with a battery

1.5.3 Application of Transformer Polarities

The polarities of the windings must be known if the transformers are connected in parallel to share a common load. Fig. 1.14a shows the parallel connection of two single-phase (1φ) transformers. This is the correct connection because secondary voltages e_{21} and e_{22} oppose each other internally.

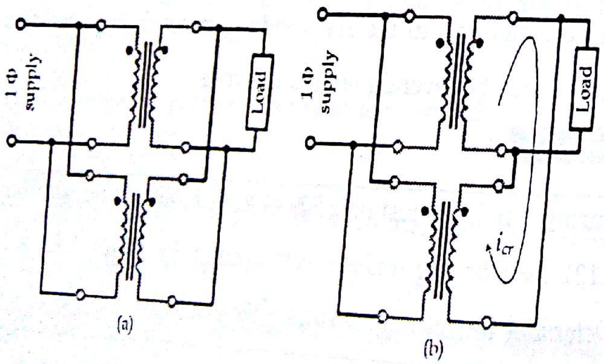


Fig. 1.14. Parallel operation of single-phase transformers (a) correct connection. (b) Wrong connection.

The connection shown in Fig. 1.14b is wrong, because e_{21} and e_{22} aid each other internally and a large circulating current i_{cir} will flow in the windings and may damage the transformers. For three-phase connection of transformers, the winding polarities must also be known.

1.6 Transformer Ratings

The transformer nameplate provides information about the ratings of the transformer. The apparent power, the voltage-handling capacity of each winding and the operating frequency are the major ratings.

The voltage ratings of a transformer serve two functions. The first one is to protect the winding insulation from the breakdown due to an excessive voltage applied to it. This is the most serious limitation in practical transformers. The second one is related to the magnetization curve and magnetizing current of the transformer.

For the sake of explanation, from the nameplate data of a 5-kVA, 500/250-V and 60-Hz single-phase transformer, we conclude that:

1. The full-load or nominal power rating of the transformer is 5 kVA. In other words, the transformer can deliver 5 kVA on a continuous basis.
2. Since it is a step-down transformer, the (nominal) primary voltage is V_1 =500 V and the (nominal) secondary voltage is V_2 = 250 V.

3. The nominal magnitudes of the primary and the secondary currents at full load are

$$I_1 = \frac{5000}{500} = 10\,A$$

$$I_2 = \frac{5000}{250} = 20\,A$$

4. Since the information on the number of turns is customarily not given by the manufacturer, we determine the *a*-ratio of the (nominal) terminal voltages as

$$a = \frac{500}{250} = 2$$

5. If this transformer is to be operated on a reduced frequency, for example 50-Hz, its applied voltage must be also reduced to 50/60 from its rating at 60-Hz or the peak flux Φ_m will be too high. This reduction in the applied voltage with frequency is called de-rating. Similarly, if the rating of the transformer is 50-Hz, the transformer may be operated at 60-Hz with a 60/50 percent higher voltages if this action does not cause insulation problems.

Review Questions (1)

- **Principles**

1. What is a transformer?
2. Can be a transformer used to transform direct voltage and direct current? Explain.
3. What differentiates a core-type transformer from a shell-type transformer? In both types, the primary and secondary windings are wrapped one on top of the other, what are the purposes of this approach?
4. Write a short note about the use and applications (importance) of the electrical transformer in our live?

- **Ideal Transformer**

5. What is (write a short note about) an ideal transformer?
6. Derive the e.m.f. Equation of a transformer.
7. Why is the frequency of the induced e.m.f. in the secondary winding of a transformer the same as that of the impressed voltage on the primary winding?
8. In a transformer, the primary current is twice as much as the secondary current. Is this a step-up or step-down transformer?

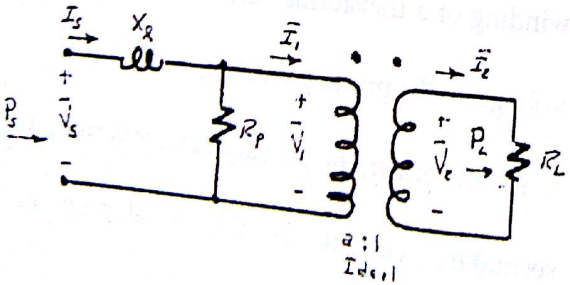
9. Explain why the primary m.m.f. must be equal and opposite of the secondary m.m.f. in an ideal transformer.

10. What is the *a*-ratio, or transformation ratio? How can the *a*-ratio be determined experimentally?

11. What is the *dot notation* of the transformer? Explain briefly two experiments can be used for determining the dot notation?

12. A distribution transformer is rated at 18 kVA, 20,000/480 V, and 60 Hz. Can this transformer safely supply 15 kVA to a 415-V load at 50 Hz? Why or why not?

13. For the ideal transformer circuit of Figure, $R_p = 18\ \Omega$, $R_L = 6\ \Omega$, and $X_\ell = 0.5\ \Omega$. If $\bar{V}_2 = 120\angle 0°$ V and $P_S = 5600$ W, (*a*) determine the turns ratio *a*, (*b*) the source voltage $\bar{V}_S$, and (*c*) the input power factor PF_S. (Ans. 2, 240.28 V, 0.999 lag)



14. For the circuit of prob. 13, $a = 10$, $R_L = 24\ \Omega$, $R_p = 3.6\ \text{k}\Omega$, $X_\ell = 100\ \Omega$, and $P_L = 2400$ W. Calculate (*a*) V_S and (*b*) P_S.

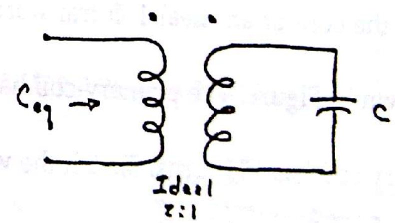
(Ans. 2405.8V, 4000W)

15. For the circuit of prob. 13, $a = 2$, $R_p = 20\ \Omega$, and $R_L = 10\ \Omega$. Determine the percentage of input power P_S that is dissipated by R_p regardless of the voltage values.

(Ans. 66.6%)

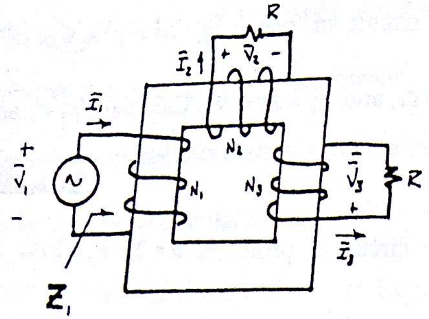
16. For the circuit of prob. 12, let $R_L = 0$, $R_p = 10\ \Omega$, $X_\ell = 1\ \Omega$, and $a = 2$. Determine the power factor P_S. (Ans. Zero-lag)

17. Determine the value of C_{eq} for the ideal transformer of the following Figure. (Ans. 0.25C)



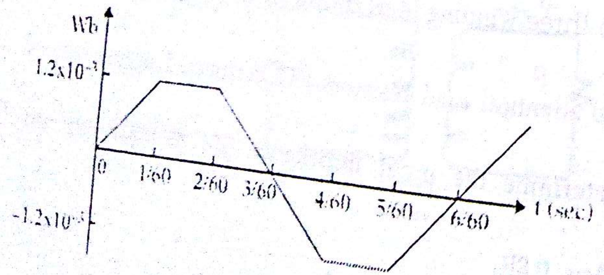
18. The three-winding ideal transformer of Figure has $N_1 = N_2 = 2N_3$ and identical load resistors (R) connected across coils 2 and 3. Determine the input impedance Z_1 as indicated on Figure.

(Ans. 0.8R)



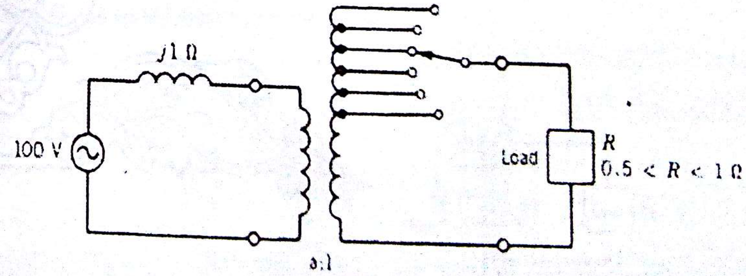
19. A 22-kVA, 2200/1100-V, step-down ideal transformer delivers a rated load at a leading power factor of 0.5. Determine (a) the secondary winding current, (b) the primary winding current, (c) the impedance on the secondary side as a parallel combination of resistance and reactance, and (d) the impedance on the primary side as a series combination of resistance and reactance.

20. The flux in the core of an ideal 1 ϕ transformer varies with time as shown in Figure. The primary coil has 400 turns and the secondary coil has 100 turns. Sketch the waveform of the induced voltage e_1 in the primary winding.



21. A resistive load varies from 1 to 0.5 Ω. The load is supplied by an ac source through an ideal transformer whose turn's ratio can be changed by using different taps as shown in the following figure. The source can be modeled as a constant voltage of 100 V (rms) in series with an inductive reactance of j1 Ω. For maximum power transfer to the load, the effective load resistance seen at the transformer primary (source side) must equal the series impedance of the generator—that is, the referred value of R to the primary side is always 1 Ω.

a) Determine the range of turns ratio for maximum power transfer to the load.
b) Determine the range of load voltages for maximum power transfer.
c) Determine the power transferred.



Ans. (a) 1.41, 1, (b) 50, 70.7 V, (c) 5000, 5000 W

Chapter 2

Practical or Real Transformers

Chapter 2

Practical or Real Transformers

2.1 Theory of Operation of Real Transformer

The ideal transformer is described in the previous chapter. However, this transformer is a hypothetical device and of course, it is never actually being made. What can be produced is a practical or real transformer. *The characteristics of a real transformer approximate that of an ideal transformer, but only to a degree.* Therefore, this section is concerned with the behavior of real transformer.

2.1.1 Effect of Leakage Flux on the voltage Ratio across a Transformer

If the supply voltage is v_1 is placed directly across the primary winding of a transformer as shown in Fig. 2.1, a time varying flux Φ_1 is established in the core.

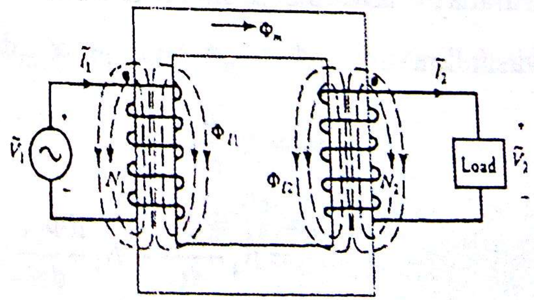


Fig. 2.1. Transformer with leakage and mutual fluxes

It can be observed from Fig. 2.1 that, not all of the flux created by the primary winding confines itself to the magnetic core on which the winding is wound. Some of the flux, known as the *leakage flux*, leaves the core and complete its path through the air. Therefore, the flux in the primary winding Φ_1 can thus be divided into two components:

a) *Mutual flux* Φ_m, which remains in the core and links both the windings, and

b) *Leakage flux* Φ_{L1}, which passes through the primary winding but returns through the air, bypassing the secondary winding:

$$\Phi_1 = \Phi_m + \Phi_{L1}$$

By the same manner, the flux in the secondary winding can be divided into:

a) *Mutual flux* Φ_m and

b) *Leakage flux* Φ_{L2}, which passes through the secondary winding but returns through the air, bypassing the primary winding:

$$\Phi_2 = \Phi_m + \Phi_{L2}$$

From Faraday's law, the primary terminal voltage can be expressed as follows:

$$v_1 = N_1 \frac{d}{dt} \Phi_1$$

$$= N_1 \frac{d\Phi_m}{dt} + N_1 \frac{d\Phi_{L1}}{dt}$$

$$= \quad e_1 \quad + \quad e_{L1}$$

where, e_1 is the primary induced emf due to the mutual flux

e_{L1} is the primary induced emf due to the leakage flux

The secondary terminal voltage of the transformer can also be expressed as follows:

$$v_2 = N_2 \frac{d}{dt} \Phi_2$$

$$= N_2 \frac{d\Phi_m}{dt} + N_2 \frac{d\Phi_{L2}}{dt}$$

$$= \quad e_2 \quad + \quad e_{L2}$$

where, e_2 is the secondary induced emf due to the mutual flux

e_{L2} is the secondary induced emf due to the leakage flux

Therefore,

$$\frac{e_1}{e_2} = \frac{N_1}{N_2} = a$$

This means that, the ratio of the primary to secondary induced emf caused by the mutual flux is equal to the turn's ratio of the transformer.

Due to the leakage flux in the primary and secondary winding, the ratio of the terminal voltages does not equal the turns ratio of the real transformer. Since in a well-designed transformer, $\Phi_m \gg \Phi_{L1}$ and $\Phi_m \gg \Phi_{L2}$, the ratio of the terminal voltages is

$$\frac{v_1}{v_2} \cong \frac{N_1}{N_2} = a$$

2.1.2 The No-Load Current in a Real Transformer

Since the core of a real transformer has a finite permeability and has a core-loss. Therefore, *even when the secondary winding is left open (no-load condition)* the primary winding draws some current, known as the *no-load* or *excitation current* I_ϕ from the source. This current is the current required to produce the flux in the core. It consists of two components:

a) The *magnetization current* I_m, which is the current necessary to produce the mutual flux in the core.
b) The *core-loss current* I_c, which is the current required to make up for hysteresis and eddy current losses in the core. If there were no core losses, component I_c would not exist, and the exciting current would reduce to only that required to establish the mutual flux.

Note that:

- *The hysteresis loss is required to accomplish the reorientation of the magnetic domains in the core during each cycle of the current applied to the core.*

- *While, the eddy current loss is the amount of energy lost due to eddy current generated in the core material. It goes into heating the iron core. There are two possible approaches to reduce these losses. The first approach is broken up the core into many small strips, or laminations, then the induced voltage in the core is reduced, resulting in a lower eddy current, and lower losses. The second approach is to increase the resistivity of the core material to reduce the amplitude of the eddy current. This is often done by adding some silicon to the steel of the core. In many cases, both approaches are combined.*

The Magnetization Current

The magnetization curve of a typical transformer core is shown in Fig. 2.2. If the flux in the transformer core is assumed to be known, then the magnitude of the magnetization current can be found directly from Fig. 2.2.

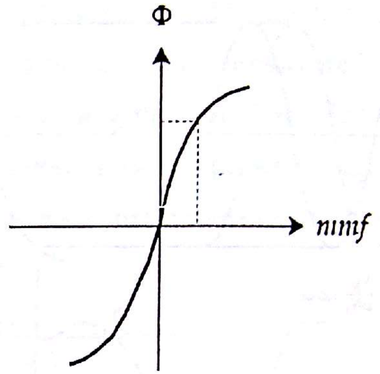


Fig. 2.2. The magnetization curve of the transformer core

By solving Faraday's law, the average flux in the core can be determined as follows:

$$\bar{\Phi} = \frac{1}{N_1}\int v_1 dt$$

If the primary voltage is given by the expression

$$v_1 = V_m \cos \omega t$$

Hence, the resulting flux is

$$\bar{\Phi} = \frac{1}{N_1}\int V_m \cos \omega t \, dt$$

$$\bar{\Phi} = \frac{V_m}{\omega N_1}\sin \omega t$$

If the values of current required to produce a given flux (Fig. 2.2) are compared to the flux in the core at different times, it is possible to construct a sketch of the magnetizing current in the winding on the core. Such a sketch is shown in Fig. 2.3.

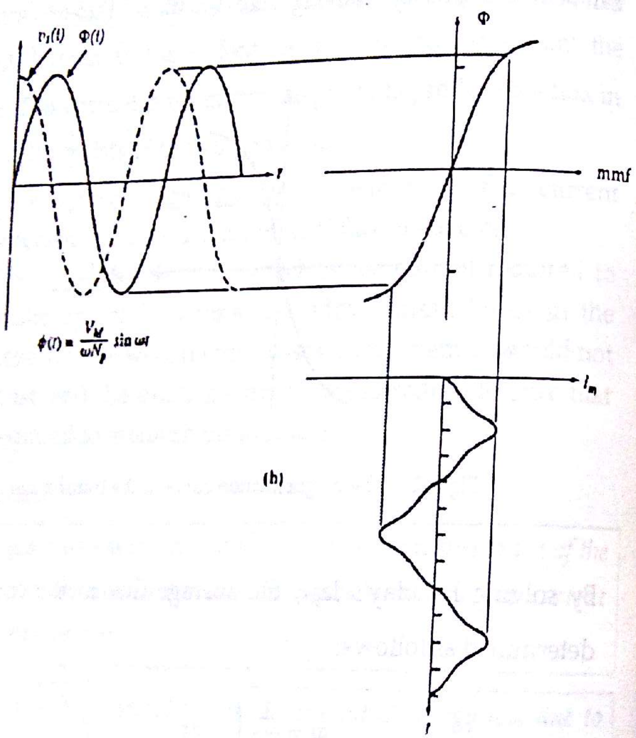


Fig. 2.3. The magnetizing current caused by the flux in the transformer core.

<u>*Note that:*</u>

- *The magnetizing current in the transformer is not sinusoidal.*
- *Once the peak flux reaches the saturation point in the core, a small increase in peak flux requires a very large increase in the peak magnetization current.*

- *The fundamental component of the magnetization current lags the voltage applied to the core by 90°.*

The Core-Loss Current

The other component of the no-load current in the transformer is the core-loss current. If the flux in the core is assumed sinusoidal, since the eddy currents in the core are proportional to $d\Phi/dt$, the eddy currents are largest when the flux in the core is passing through 0 Web. Therefore, the core-loss current is greatest as the flux passes through zero. The total current required to make up for core losses is shown in Fig. 2.4.

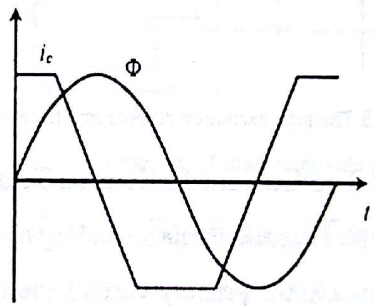


Fig. 2.4. The core-loss current in a transformer

<u>*Note that:*</u>

- *The core-loss current is nonlinear because of the nonlinear effects of hysteresis.*
- *The fundamental component of the core-loss current is in phase with the voltage applied to the core.*

From the above discussion, the total no-load or excitation current in the core of the transformer is just the sum of the magnetization current and the core-loss current:

$$i_\phi = i_m + i_c$$

Therefore, the waveform of the total excitation current in a typical transformer core is the sum of the magnetization and core-loss currents given in Figs. 2.3 and 2.4, this result in the waveform of Fig. 2.5.

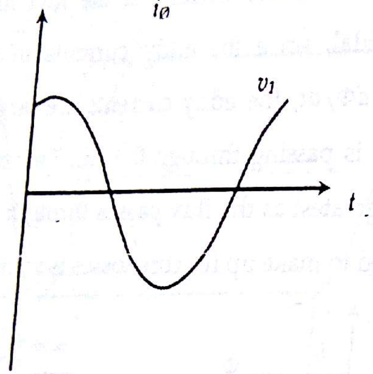


Fig. 2.5. The total excitation current in a transformer

2.1.3 The Current Ratio on a Transformer

When a load is connected to the secondary of the transformer as shown in Fig. 2.6, the primary current produces a positive $mmf_1 = N_1 i_1$ and the secondary current produces a negative $mmf_2 = -N_2 i_2$. Therefore, the net mmf that produce the net flux in the core must be

$$mmf_{net} = N_1 i_1 - N_2 i_2 = \Phi\Re$$

where $\Re$ is the reluctance of the transformer core. Note that, the net mmf in the ideal transformer is zero.

Because the reluctance of a well-designed transformer core is very small (nearly zero) until the core is saturated, the relationship between the primary and secondary currents is approximately

$$mmf_{net} = N_1 i_1 - N_2 i_2 \cong 0$$

Therefore,

$$N_1 i_1 \cong N_2 i_2$$

$$\frac{i_1}{i_2} \cong \frac{N_2}{N_1} = \frac{1}{a}$$

Note that:

- *The mmf in the core or real transformer is nearly zero.*

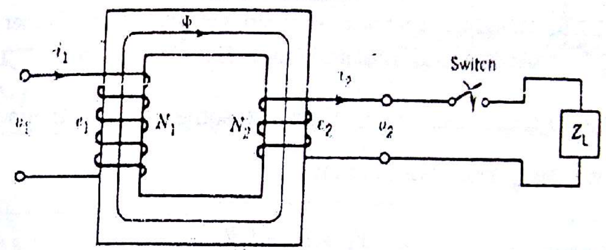


Fig. 2.6. Transformer with load

2.2 Transformer Equivalent Circuits

In the previous section, we placed quite a few characteristics to obtain useful definitions about the real transformer. In this section, our aim is to use those characteristics in order to develop an equivalent circuit for a real transformer.

2.2.1 Considering the Winding Resistances

In the ideal transformer, no winding resistance is considered. Nevertheless, the real transformer has primary and secondary winding resistances. Therefore, the real transformer can be modeled by an ideal transformer, including a lumped resistance equal to the winding resistance of series with each winding as

shown in Fig. 2.7, where R_1 and R_2 are the winding resistances of the primary and the secondary, respectively.

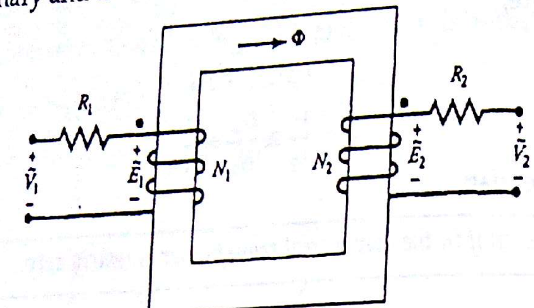


Fig. 2.7. Real transformer represented in terms of an ideal transformer with winding resistances modeled as lumped resistances.

Due to these resistances, there are some voltage drops in the two winding. The result is that:

$$V_1 = E_1 + I_1R_1$$

$$V_2 = E_2 - I_2R_2$$

Note that:

The inclusion of the winding resistances dictates that
(a) The power input must be greater than the power output,
(b) The terminal voltage is not equal to the induced emf, and
(c) The efficiency (the ratio of power output to power input) of a real transformer is less than 100%.

2.2.2 Considering the Leakage Fluxes

From the discussion given in section 2.1.1, not all of the flux created by a winding confines itself to the magnetic core on which the winding is wound. Part of the flux, known as the leakage flux, does complete its path through the air. Therefore, when both windings in a transformer carry currents, each creates

its own leakage flux, as illustrated in Fig. 2.1. Although a leakage flux is a small fraction of the total flux created by a winding, it does affect the performance of a transformer.

The leakage flux associated with either winding is responsible for the voltage drop across it. Therefore, we can represent the voltage drop due to the leakage flux by a leakage reactance.

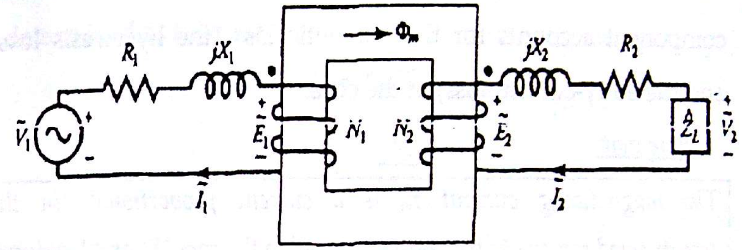


Fig. 2.8. Real transformer represented in terms of an ideal transformer with winding resistances and leakage reactances.

If X_1 and X_2 are the leakage reactances of the primary and secondary windings, a real transformer can then be represented in terms of an ideal transformer with winding resistances and leakage reactances as shown in Fig. 2.8.

Due to these resistances and reactances, the voltage relations in the two winding are as follows:

$$V_1 = E_1 + I_1(R_1 + jX_1)$$

$$V_2 = E_2 - I_2(R_2 + jX_2)$$

where

$$\frac{E_1}{E_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2} = a$$

2.2.3 Considering the Finite Permeability

Since the core of a real transformer has a finite permeability and has a core loss. Therefore, even when the secondary is left open (no-load condition) the primary winding draws the excitation current, from the source. As mentioned before, the excitation current I_ϕ is the sum of two currents: the core-loss current I_c and the magnetizing current I_m. The core-loss component accounts for the magnetic loss (the hysteresis loss and the eddy-current loss) in the core.

Note that:

The magnetizing current I_m is a current proportional (in the unsaturated region) to the voltage applied to the core 'E' and lagging it by 90°, so it can be modeled by a reactance X_m connected in parallel with induced emf.

The core-loss current (I_c) is a current proportional to the voltage applied to the core that is in-phase with the applied voltage, so it can be modeled by an equivalent resistance (R_c) connected in parallel with the induced emf.

Remember that:

Both magnetizing and core-loss currents are nonlinear, so the inductance X_m and the resistance R_c are, at best, approximations of the real excitation effects.

Therefore, the equivalent circuit of Fig. 2.8 is modified to include the core-loss resistance R_c and the magnetizing reactance X_m. Such a circuit is shown in Fig. 2.9.

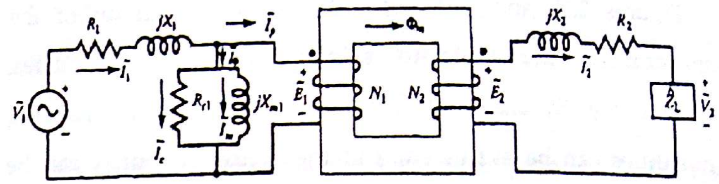


Fig. 2.9. Equivalent circuit of a transformer, including winding resistances, leakage reactance, core-loss resistance, and magnetizing reactance.

Then, the core-loss and magnetizing currents can be determined as follows:

$$I_c = E_1/R_{c1}$$

$$I_m = \frac{E_1}{jX_{m1}}$$

Notice that:

The elements forming the excitation branch R_c and X_m are placed in the primary side. This is because the induced emf actually applied to the core is approximately equal to the input voltage to the transformer.

2.2.4 Referred Equivalent Circuit of Single-Phase Transformer

In order to analyze the equivalent circuit containing transformers, it is normally necessary to convert the entire circuit to an equivalent circuit at a single voltage level. Therefore, the ideal transformer in the equivalent circuit of Fig. 2.9 can be moved to the right or left by referring all quantities to the primary or secondary side, respectively. For convenience, the ideal transformer is usually not shown and the equivalent circuit is drawn.

Figures 2.10 and 2.11 show the equivalent circuit of the transformer referred to its primary and secondary sides, respectively. By analyzing this equivalent circuit, the referred quantities can be evaluated, and the actual quantities can be determined from them if the turn's ratio is known.

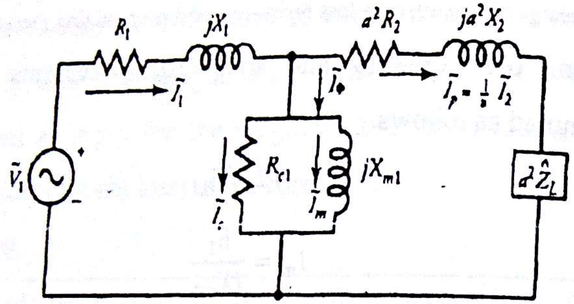


Fig. 2.10. The exact equivalent circuit as viewed from the primary side

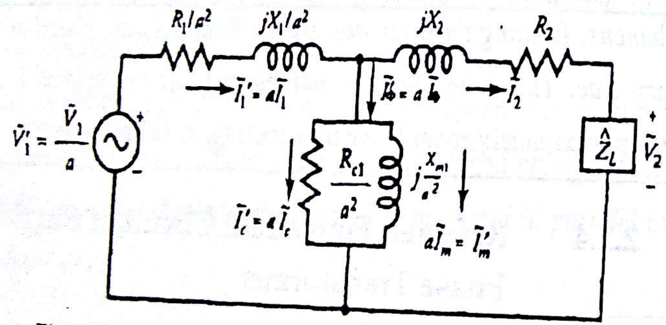


Fig. 2.11. The exact equivalent circuit as viewed from the secondary side of the transformer.

Phasor Diagram

If the single-phase transformer operates under steady-state conditions, the values of currents, voltages, and phase angles can be obtained by its phasor diagram.

Note that:

In the transformer phasor diagram, the load voltage is used as a reference because quite often it is a known quantity.

Let V_2 be the voltage across the load impedance Z_L, I_2 is the load current and θ_2 is the angle between the load voltage and current. Depending upon Z_L, I_2 may be leading, in-phase, or lagging with V_2.

Figure 2.12 shows the phasor diagrams of the exact equivalent circuit of a single-phase transformer referred to the primary side at a lagging power factor.

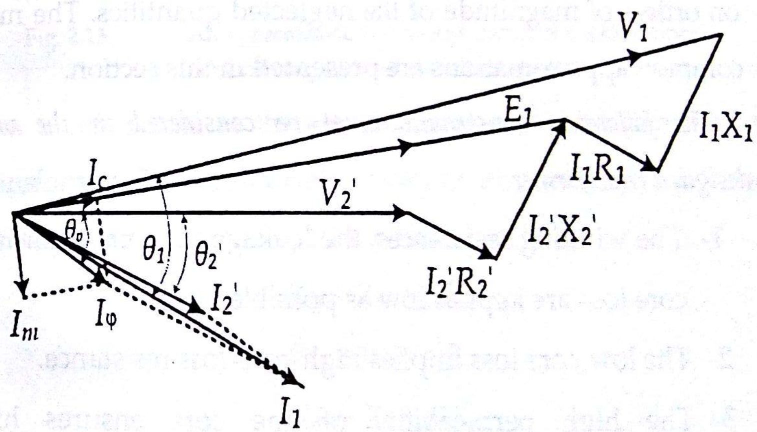


Fig. 2.12. The phasor diagram of an exact equivalent circuit of a single-phase transformer referred to the primary side of the lagging power factor

Exercise:

Phasor diagrams for the exact equivalent circuits at unity power factor and leading power factor can also be drawn and are left as exercises for the student.

2.2.5 Approximate Equivalent Circuits of Single-Phase Transformer

The *exact equivalent circuit* of the transformer shown before is often *more complex* than necessary in order to get good results in practical engineering applications.

In engineering analyses involving the transformer as a circuit element, it is customary to adopt one of several approximate forms of the equivalent circuit of Figs. 2.13 and 2.14 rather than the full circuit. The approximations chosen in a particular case depend largely on physical reasoning based on orders of magnitude of the neglected quantities. The more common approximations are presented in this section.

The following constraints must be considered in the well-designed transformer:

1- The winding resistances, the leakage reactances, and the core loss are kept as low as possible.

2- The low core loss implies high core-loss resistance.

3- The high permeability of the core ensures high magnetizing reactance.

Thus, the impedance of the so-called parallel branch (R_c in parallel with jX_m) across the primary is very high compared with $Z_1 = R_1 + jX_1$ and $Z_2 = R_2 + jX_2$. The high impedance of the parallel branch assures low excitation current.

Since Z_i is kept low, the voltage drop across it is also low in comparison with the applied voltage.

Therefore, without introducing any appreciable error in our calculations, it can be assumed that, the voltage drop across the parallel branch is the same as the applied voltage. *This assumption allows us to move the parallel branch as indicated in Fig. 2.13 for the equivalent circuit of a transformer embodying an ideal transformer.*

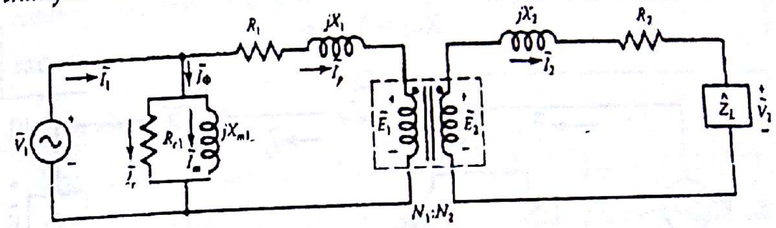


Fig. 2.13. An approximate equivalent circuit of a transformer embodying an ideal transformer.

This is referred to as the **approximate equivalent circuit** of a transformer. The *approximate equivalent circuit as referred to the primary side* is given in Fig. 2.14, where

$$Z_{e1} = R_{e1} + jX_{e1}$$

$$R_{e1} = R_1 + a^2R_2$$

$$X_{e1} = X_1 + a^2X_2$$

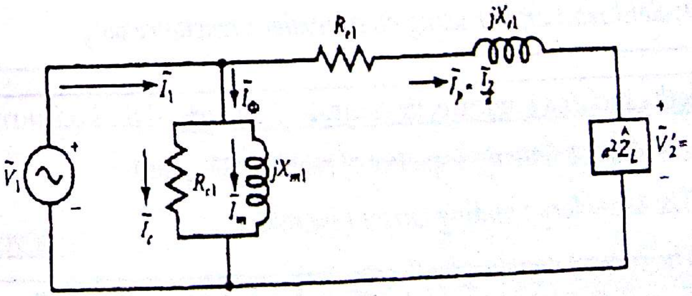


Fig. 2.14. The approximate equivalent circuit of a transformer referred to the primary side.

Similarly, Fig. 2.15 shows the *approximate equivalent circuit as referred to the secondary side* of the transformer, where

$$Z_{e2} = R_{e2} + jX_{e2}$$

$$R_{e2} = R_2 + R_1/a^2$$

$$X_{e2} = X_2 + X_1/a^2$$

$$R_{c2} = R_c/a^2$$

$$X_{m2} = X_m/a^2$$

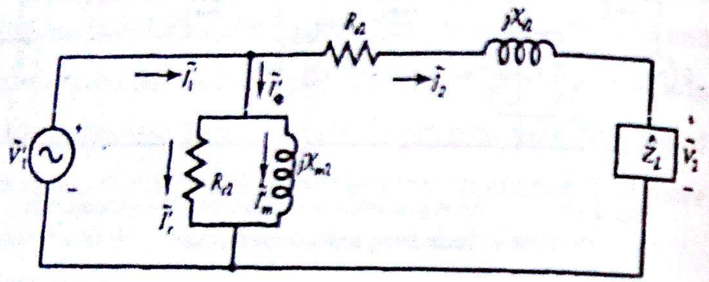


Fig. 2.15. The approximate equivalent circuit of a transformer referred to the secondary side.

Note that:

In the analysis of complex power systems, a great deal of simplification can be assumed by neglecting the excitation current and the transformer is modeled using equivalent resistance in series with an equivalent reactance or using an equivalent reactance only.

In both exact and approximate equivalent circuits, if the load current is increased, the following sequence of events takes place:

a) The secondary winding current increases.

b) The current supplied by the source increases.

c) The voltage drops across the primary winding impedance increases.

d) The induced emf drops.

e) Finally, the mutual flux decreases owing to the decrease in the magnetizing current.

The decrease in the mutual flux in the well-designed transformer from no load to full load is about 1% to 3%. Therefore, for all practical purposes, we can assume that E_1 remains substantially the same. In other words, the mutual flux is essentially the same under normal loading conditions and thereby there is no appreciable change in the excitation current.

Phasor Diagram

The phasor diagrams of the approximately equivalent circuit of a single-phase transformer referred to the primary side at lagging power factor is shown in Fig.2.12.

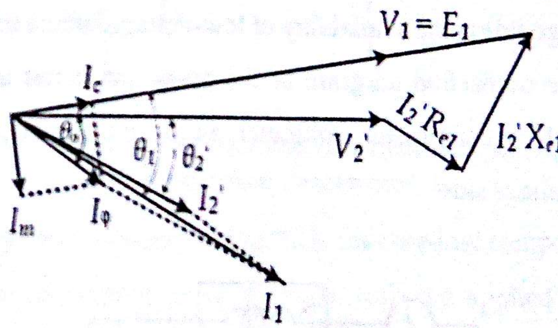


Fig. 2.16. The phasor diagram of an approximately equivalent circuit of a transformer referred to the primary side of the lagging power actor

Exercise:

Phasor diagrams for the approximately equivalent circuits at unity power factor and leading power factor can also be drawn and are left as exercises for the student.

2.3 Determination of Transformer Equivalent Circuit Parameters

The transformer equivalent-circuit parameters can be directly determined by performing the following tests.

1. Open circuit or no-load test
2. Short circuit test

2.3.1 Open-Circuit Test

In this test, one winding of the transformer is left open while the other is excited by applying the *rated voltage and rated frequency*. Although it does not matter which side of the transformer is excited, it is safer to conduct the test on the *low-voltage side*. Another reason for performing the test on the low-voltage side is the availability of low-voltage source in the lab.

The connection diagram of the open-circuit test is shown in Fig. 2.17 with ammeter, voltmeter, and wattmeter inserted on the low-voltage side.

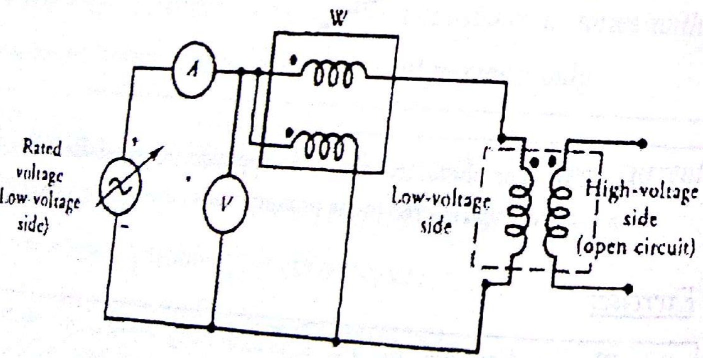


Fig. 2.17. A two-winding transformer wired with instruments for open-circuit test.

Three readings are available from the measuring devices in this test;

a) The voltmeter reading

$V_{o.c} = V_L$ *(rated voltage at the low voltage side),*

b) The ammeter reading

$I_{o.c} = I_\phi$ *(no-load or excitation current)*

c) The wattmeter reading

$P_{o.c} = P_c$ *(no-load or core-loss power)*

The approximate equivalent of the transformer in the open circuit test referred to the low-voltage side is shown in Fig. 2.18.

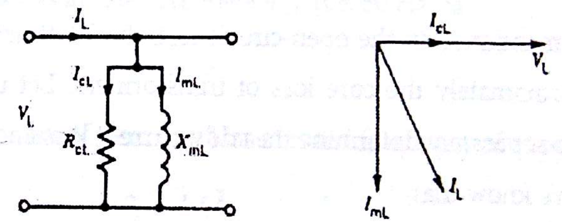


Fig. 2.18. Approximately equivalent circuit and its phasor diagram of the transformer for open circuit test.

From the phasor diagram of Fig. 2.18, the core-loss component of the excitation current I_{cL} is in phase with the applied voltage while the magnetizing current I_{mL} lags the applied voltage by 90°. Therefore, the power factor at this test $\cos\theta_{o.c}$ is *lag p.f.* and it can be calculated from the test readings as follows:

$$\cos\theta_{o.c} = \frac{P_{o.c}}{V_{o.c}\cdot I_{o.c}}$$

$$\theta_{o.c} = \cos^{-1}\frac{P_{o.c}}{V_{o.c}\cdot I_{o.c}}$$

The core-loss and magnetizing currents are

$$I_{cL} = I_{o.c} \cos \theta_{o.c}$$

and

$$I_{mL} = I_{o.c} \sin \theta_{o.c}$$

Thus, the core-loss resistance and the magnetizing reactance as viewed from the

$$R_{cL} = \frac{V_{o.c}}{I_{cL}} = \frac{V_{o.c}^2}{P_{o.c}}$$

$$X_{mL} = \frac{V_{o.c}}{I_{mL}}$$

Determination of eddy current loss and hysteresis loss in transformer

As mentioned in the open-circuit test, the Wattmeter reading is approximately the core loss of transformers. Let us now see how to separately determine the eddy current loss and hysteresis loss. We know that

$$P_h = \text{hysteresis loss} \propto (B_m)^{1.6} f$$

$$P_e = \text{eddy current loss} \propto (B_m)^2 f^2$$

$$P_c = \text{core loss} = P_h + P_e$$

$$P_c = k_h (B_m)^{1.6} f + k_e (B_m)^2 f^2$$

$$P_c = af + bf^2$$

where a and b are constants.

Therefore,

$$P_c/f = a + bf$$

Therefore, if we vary f and plot P_c/f against f, then we will get a straight-line curve PQ as shown in Fig. 2.19. From this figure, the length of line $OP = a$ and $\tan\theta = b$

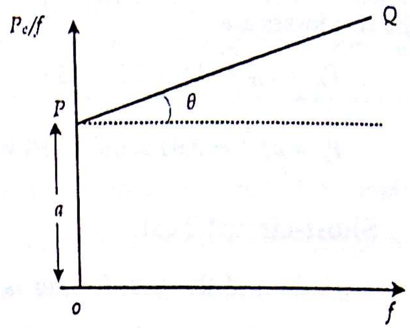


Fig. 2.19. Curve of P_c/f against f

Thus, the constant 'a' and 'b' are determined. Therefore, the hysteresis loss at rated frequency f_r (i.e. 50 Hz) is

$$P_h = af_r$$

Similarly, the eddy current loss at rated frequency of supply is

$$P_e = bf_r^2$$

Example

In a transformer, the core loss is found to be 52 W at 40 Hz and 90 W at 60 Hz measured at same peak flux density. Compute the hysteresis and eddy current losses at 50 Hz.

Solution

Since the flux density is the same in both cases, we can use the relation

$$P_c/f = a + bf$$

$$\therefore 52/40 = a + 40b \text{ and}$$

$$90/60 = a + 60b$$

$\therefore a = 0.9$ and $b = 0.01$

At 50 Hz, the two losses are

$$P_h = af_r = 0.9 \times 50 = 45\ W$$

$$P_e = bf_r^{\,2} = 0.01 \times 50^2 = 25\ W$$

2.3.2 Short-circuit Test

In this test, one winding of the transformer is short-circuited while the other is connected to a fairly low voltage source. The applied voltage is carefully adjusted so that each winding carries a rated current. The rated current in each winding ensures a proper simulation of the leakage flux pattern associated with that winding. (Be sure to keep the applied voltage and current at a safe level. *It would not be a good idea to burn out the transformer windings while trying to test it*). The input voltage, current, and power are again measured as shown in the connection diagram shown in Fig. 2.20.

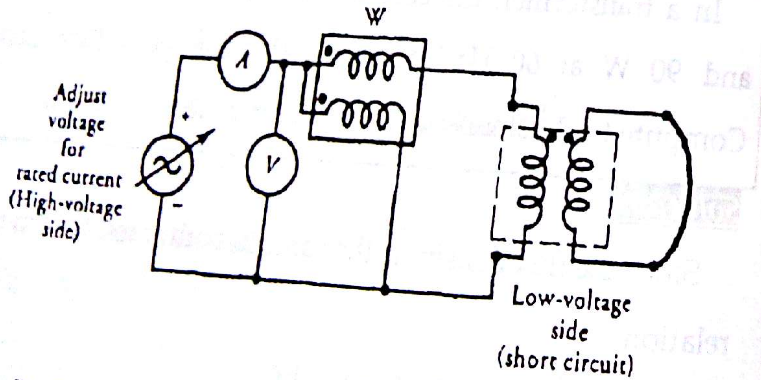


Fig. 2.20. A two-winding transformer wired for short-circuit test.

Similarly, with the open-circuit test, it does not really matter on which side this test is performed. However, the measurement

of the rated current suggests that, for safety purposes, the test be performed on the high-voltage side (i.e. low-voltage side is short-circuited). The test arrangement with all instruments inserted on the high-voltage side with a short circuit on the low voltage side is shown in Fig. 2.20.

Since the applied voltage required to maintain the rated currents is a small fraction of the rated voltage, both the core-loss and the magnetizing current components are so small that they can be neglected. This means, the core loss is assumed zero.

Three readings are available from the measuring devices in this test;

a) The voltmeter reading

$V_{s.c}$ = *about from 10:15 percent of rated voltage at high voltage side,*

b) The ammeter reading

$I_{s.c} = I_H$ *(equals the rated current in the high voltage side)*

c) The wattmeter reading

$P_{s.c} = P_{cu}$ *(equals the full-load copper losses)*

The approximate equivalent circuit for a short-circuit test as viewed from the high-voltage side is given in Fig. 2.21.

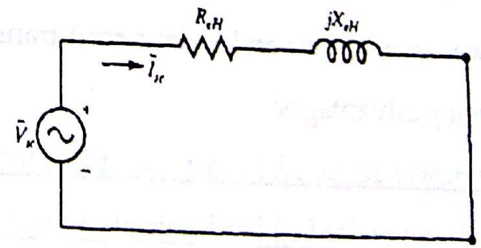


Fig. 2.21. Approximately equivalent circuit of the transformer for short circuit test.

From the short-circuit test readings, the parameter of the equivalent circuit shown in Fig. 2.21 can be determined as follows:

- *The total resistance of the two windings as referred to the high-voltage side.*

$$R_{eH} = \frac{P_{s.c}}{I_{s.c}^2}$$

- *The total impedance of the two windings as referred to the high-voltage side.*

$$Z_{eH} = \frac{V_{s.c}}{I_{s.c}}$$

- *So that, the total leakage reactance of the two windings as referred to the high voltage side is*

$$X_{eH} = \sqrt{(Z_{eH})^2 - (R_{eH})^2}$$

For optimum design criterion, the copper loss on the high-voltage side is equal to the copper loss on the low-voltage side. Under this criterion

$$R_H = 0.5R_{eH}$$

Similarly, we can assume that

$$X_H = 0.5X_{eH}$$

2.3.3 Advantages of Transformer Tests

The two, open-circuit and short-circuit transformer tests offer the following advantages:

1. The power required to carry out these tests is very small as compared to the full-load output of the transformer. In case of open-circuit lest, the power required is equal to the iron loss, whereas for a short-circuit test, the power required is equal to full-load copper loss.

2. These tests enable us to determine the efficiency of the transformer accurately at any load and p.f. without actually loading the transformer.
3. The short-circuit test enables us to determine R_{e1} and X_{e1}. We can thus find the total voltage drop in the transformer as referred to primary or secondary. This permits us to calculate voltage regulation of the transformer.

2.3.4 Why Transformer Rating in kVA?

An important factor in the design and operation of electrical machines is the relation between *the life of the insulation* and *operating temperature of the machine*. Therefore, temperature rise resulting from the losses is a determining factor in the rating of a machine. Since the copper loss in a transformer depends on current and iron loss depends on voltage. *Therefore*, the total loss in a transformer depends on the volt-ampere product only and not on load power factor. For this reason, the rating of a transformer is in kVA and not kW.

EXAMPLE 2.1

Tests are performed on a single-phase, 10 kVA, 2200/220 V, 60 Hz transformer and the following results are obtained.

	Open-Circuit Test (high-voltage side open)	Short-Circuit Test (low-voltage side shorted)
Voltmeter	220 V	150 V
Ammeter	2.5 A	4.55 A
Wattmeter	100 W	215 W

a) Derive the parameters for the approximate equivalent circuits referred to the low-voltage side and the high-voltage side.

b) Express the excitation current as a percentage of the rated current.

c) Determine the power factor for the no-load and short-circuit tests.

Solution

Note that for the no-load test the supply voltage (full-rated voltage of 220V) is applied to the low-voltage winding, and for the short-circuit test the supply voltage is

applied to the high-voltage winding with the low-voltage winding shorted. The ratings of the windings are as follows:

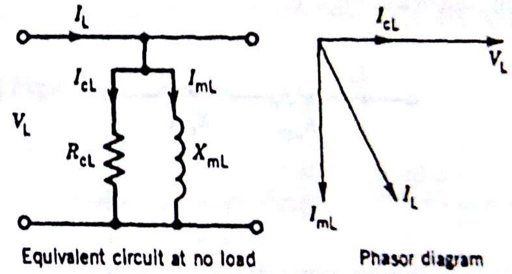
$$V_{1(rated)} = 2200\ V$$

$$V_{2(rated)} = 220\ V$$

$$I_{1(rated)} = \frac{10000}{2200} = 4.55\ A$$

$$I_{2(rated)} = \frac{10000}{220} = 45.5\ A$$

(a) The equivalent circuit and the phasor diagram for the open-circuit test are shown in the following Figure.



Equivalent circuit at no load

Phasor diagram

$$\text{Power, } P_{OC} = \frac{V_2^2}{R_{c2}}$$

$$\text{Then } R_{c2} = \frac{220^2}{100} = 484\ \Omega$$

$$I_{c2} = \frac{220}{484} = 0.45\ A$$

$$I_{m2} = \sqrt{\left(I_2^2 - I_{c2}^2\right)} = \sqrt{\left(2.5^2 - 0.45^2\right)} = 2.46 A$$

$$X_{m2} = \frac{V_2}{I_{m2}} = \frac{22}{2.46} = 89.4\ \Omega$$

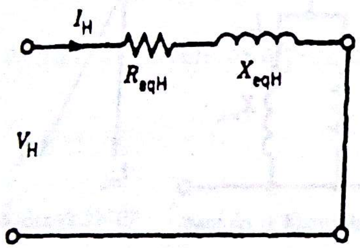
The corresponding parameters for the high-voltage side are obtained as follows:

Turns ratio $a = \frac{2200}{220} = 10$

$$R_{c1} = a^2 R_{c2} = 10^2 * 484 = 48\,400\ \Omega$$

$$X_{m1} = a^2 X_{m2} = 10^2 * 89.4 = 8940\ \Omega$$

The equivalent circuit with the low-voltage winding shorted is shown in the following Figure.



Equivalent circuit for short-circuited low-voltage winding

Power $P_{sc} = I_1^2 R_{eq1}$

Then, $R_{eq1} = \frac{215}{4.55^2} = 10.4\ \Omega$

$$Z_{eq1} = \frac{V_{sc1}}{I_{sc1}} = \frac{150}{4.55} = 32.97\ \Omega$$

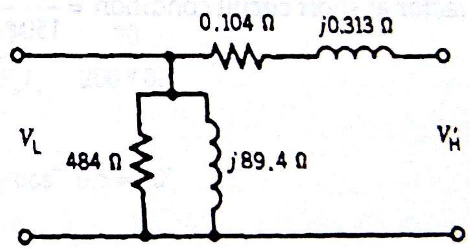
Then, $X_{eq1} = \sqrt{Z_{eq1}^2 - R_{eq1}^2} = 31.3\ \Omega$

The corresponding parameters for the low-voltage side are as follows:

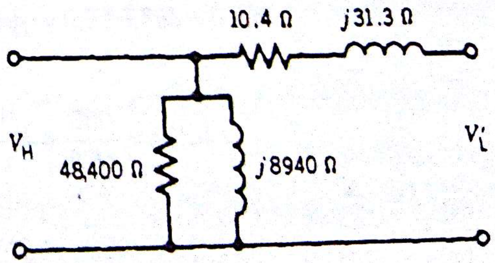
$$R_{eq2} = \frac{R_{eq1}}{a^2} = \frac{10.4}{10^2} = 0.104\ \Omega$$

$$X_{eq2} = \frac{X_{eq1}}{a^2} = \frac{31.3}{10^2} = 0.313\ \Omega$$

The approximate equivalent circuits referred to the low-voltage side and the high-voltage side are shown in the following Figure.



Referred to low-voltage side



Referred to high-voltage side

Note that the impedance of the shunt branch is much larger than that of the series branch.

(b) From the no-load test the excitation current, with rated voltage applied to the low-voltage winding, is:

$$I_o = 2.5A$$

This is $\frac{2.5}{45.5} * 100\% = 5.5\%$ of the rated current of the winding

(c) power factor at no load $= \frac{Power}{volt \ ampere}$

$$= \frac{100}{220 * 2.5} = 0.182$$

$$\text{Power factor at short circuit condition} = \frac{215}{150 * 4.55} = 0.315$$

EXAMPLE 2.2

Obtain the equivalent circuit of a 200/400-V, 50 Hz, a single-phase transformer from the following test as follows :-

O.C. test : 200 V, 0.7 A, 70W-on LV side

S.C. test : 15 V, 10 A, 85 W-on HV side

Calculate the secondary voltage when delivering 5 kW at 0.8 power factor lagging, the primary voltage being 200 V.

Solution

From O.C. Test

$$P_o = V_o I_o * \cos\varphi_o$$

$$\therefore \cos\varphi_o = \frac{P_o}{V_o I_o} = \frac{70}{200 * 0.7} = 0.5$$

then $\varphi_o = \cos^{-1} 0.5 = 60°$

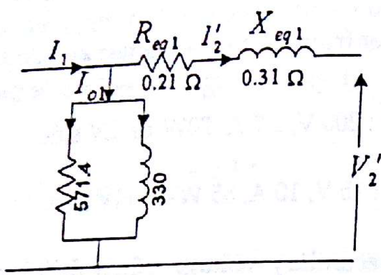
then $I_{c1} = I_o \cos\varphi_o = 0.7 * 0.5 = 0.35A$

$$I_{m1} = I_o \sin\varphi_o = 0.7 * 0.866 = 0.606A$$

then $R_{c1} = \frac{V_{o1}}{I_{c1}} = \frac{200}{0.35} = 571.4\ \Omega$

and $X_{m1} = \frac{V_{o1}}{I_{m1}} = \frac{200}{0.606} = 330\ \Omega$

As shown in the following Figure, these values refer to primary i.e. low-voltage side



From Short Circuit test:

It may be noted that in this test instruments have been placed in the secondary i.e. high-voltage winding and the low-voltage winding i.e. primary has been short-circuited.

Now, $Z_{eq2}=\dfrac{V_{2sc}}{I_{2sc}}=\dfrac{15}{10}=1.5\Omega$

$$Z_{eq1}=a^2*Z_{eq2}=\left(\frac{1}{2}\right)^2*1.5=0.375\Omega$$

Also, $P_{sc}=I_{2sc}^2\ R_{eq2}$

Then, $R_{eq2}=\dfrac{85}{100}=0.85\Omega$

Then, $R_{eq1}=a^2*R_{eq2}=\left(\dfrac{1}{2}\right)^2*0.85=0.21\ \Omega$

Then, $X_{eq1}=\sqrt{Z_{eq1}^2-R_{eq1}^2}=\sqrt{0.375^2-0.21^2}=0.31\ \Omega$

Output KVA $=\dfrac{real\ power}{Power\ factor}=\dfrac{5}{0.8}=6.3\ kVA$

Output current $I_2=\dfrac{5000}{0.8*400}=15.6A$

Now, from the aproximate equivalent circuit refeared to secondery : $V_2\ \angle 0^\circ=V_1'\ \angle\delta^\circ-I_2\angle\ \varphi^\circ*Z_{eq2}$

Then, $V_2\angle\underline{0^\circ}=400\angle\delta^\circ-15.6\angle-36.87^\circ*(0.85+j1.2)$

$$V_2\angle 0^\circ=400\angle\delta^\circ-15.6\angle-36.87^\circ*1.5\angle\ 54.7^\circ$$

$$V_2\angle 0^\circ=400\angle\delta^\circ-23.4\angle 18.17^\circ$$

From the above equation we have two unknown variables $V_2\ and\ \delta^\circ$ it need two equations to get both of them. The above equation is a complex one so we can get two equations out of it. If we equate the real parts together and the equate the imaginary parts:

So from the Imaginary parts:

$$|V_2|\sin(0) = 400\sin(\delta^{o}) - 23.4*\sin(18.17^{o})$$

$$0 = 400*\sin(\delta^{o}) - 7.41^{o}$$

Then, $\delta^{o} = 7.4^{o}$

So from the Real parts:

$$|V_2|\cos(0) = 400*\cos(7.41^{o}) - 23.4*\cos(18.17^{o})$$

Then, $|V_2| = 374.5\ V$

EXAMPLE 2.3

A 50 Hz, $1-\phi$ transformer has a turns ratio of 6. The resistances are 0.9 Ω, 0.03 Ω and reactances are 5Ω and 0.13 Ω for high-voltage and low-voltage, windings respectively. Find (a) the voltage to be applied to the HV side to obtain full-load current of 200 A in the LV winding on short-circuit (b) the power factor on short-circuit.

Solution

The turn's ratio is $a = \frac{N_1}{N_2} = 6$

$$R_{eq1} = R_1 + a^2 R_2 = 0.9 + 6^2 * 0.03 = 1.98\ \Omega$$

$$X_{eq1} = X_1 + a^2 X_2 = 5 + 6^2 * 0.13 = 9.68\ \Omega$$

$$Z_{eq1} = \sqrt{R_{eq1}^2 + X_{eq1}^2} = \sqrt{1.98^2 + 9.68^2} = 9.88\ \Omega$$

$$I_1 = \frac{I_2}{a} = \frac{200}{6} = 33.33A$$

(a) $V_{sc} = I_1 * Z_{eq1} = 9.88 * 33.33 = 329.3V$

(b) $\cos\varphi = \frac{R_{eq1}}{Z_{eq1}} = \frac{1.98}{9.88} = 0.2$

EXAMPLE 2.4

A 1-phase, 10 kVA, ,500/250-V, 50 Hz transformer has the following constants:

Resistance: Primary 0.2 Ω; .Secondary 0.5Ω

Reactance: Primary 0.4Ω ; Secondary 0.1 Ω

Resistance of equivalent exciting circuit referred to primary, $R_{c1} = 1500\Omega$

Reactance of equivalent exciting circuit referred to primary, $X_{m1} = 750\ \Omega$.

What would be the readings of the instruments when the transformer is connected for the open-circuit and-short-circuit tests?

Solution

O.C. Test:

$$I_{m1} = \frac{V_1}{X_m} = \frac{500}{750} = \frac{2}{3} A$$

$$I_{c1} = \frac{V_1}{R_{c1}} = \frac{500}{1500} = \frac{1}{3} A$$

$$I_o = \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2} = 0.745\ A$$

No load primary input

$$V_1 * I_{c1} = 500 * \frac{1}{3} = 167\ W$$

Instruments used in primary circuit are: voltmeter, ammeter and wattmeter, their readings being 500 V, 0745 A and 167 W respectively.

S.C. Test

Suppose S.C. test is performed by short-circuiting the LV, winding i.e. the secondary so that all instruments are in primary.

$$R_{eq1} = R_1 + R_2' = R_1 + a^2 R_2 = 0.2 + 4 * 0.5 = 2.2\ \Omega$$

$$X_{eq1} = X_1 + X_2' = X_1 + a^2 X_2 = 0.4 + 4 * 0.1 = 0.8\Omega$$

Then, $Z_{eq1} = \sqrt{R_{eq1}^2 + X_{eq1}^2} = \sqrt{2.2^2 + 0.8^2} = 2.341\ \Omega$

Full-load primary current

$$I_1 = \frac{Rated\ kVA}{Rated\ Primary\ voltage} = \frac{10000}{500} = 20A$$

Then $V_{sc} = I_1 * Z_{eq1} = 20 * 2.431 = 46.8V$

Power absorbed $= I_1^2 * R_{eq1} = 20^2 * 2.2 = 880W$

Primary instruments will read: 468 V, 20 A, 880 W.

2.4 Efficiency

All devices or machines in the electric power system are desired to operate at a high efficiency. The percentage efficiency η of a device is defined as the ratio of output power, P_{out} to the input power, P_{in} as follows:

$$\%\,\eta = \frac{P_{out}}{P_{in}} \times 100$$

The input power is the sum of output power and the power losses, P_{loss}. Thus

$$\%\,\eta = \frac{P_{out}}{P_{out} + P_{loss}} \times 100$$

Fortunately, losses in transformers are small. Because the transformer is a static device, there are no rotational losses such as windage and friction losses in a rotating machine. In a well-designed transformer the efficiency can be nearly of the ideal transformer.

The output power is given by

$$P_{out} = V_2 I_2 \cos\theta_2$$

$$= V_2' I_2' \cos\theta_2$$

There are two types of losses present in real transformers:

1. *Copper loss*, P_{cu}: These losses are accounted by the winding resistances R_1 and R_2 or R_e in the equivalent circuits. It varies as the square of the current in each winding. Therefore, as the load increases, the copper loss is increased. That is why the copper loss is also termed the *variable loss*.

2. Core or Magnetic loss, P_c: This loss consists of eddy-current and hysteresis losses. For a given flux density and the frequency of operation, the eddy-current loss can be minimized by using thinner laminations. On the other hand, the hysteresis loss depends upon the magnetic characteristics of the type of steel used for the magnetic core. Since the flux, Φ_m in the core of a transformer is practically constant for all conditions of load, the core (magnetic) loss, P_c is essentially constant. It is for this reason that the core loss is often referred to as the *fixed loss*. These losses are determined on the equivalent core resistance R_C ($P_C=E^2/R_C$).

Therefore, the efficiency of the transformer can be expressed by

$$\%\,\eta = \frac{V_2' I_2' \cos\theta_2}{V_2' I_2' \cos\theta_2 + P_c + {I_2'}^2 R_{e1}} \times 100$$

In general, this equation is modified in order to determine the efficiency of the transformer at any percentage of loading as follows:

$$\%\,\eta = \frac{x \cdot S_{out}^{F.L} \cos\varphi}{x \cdot S_{out}^{F.L} \cos\varphi + P_c + x^2 \cdot P_{cu}^{F.L}} \times 100$$

where x is the per unit loading
$S_{out}^{F.L}$ is the full-load apparent power
$P_{cu}^{F.L}$ is the full-load copper loss power

The p.u loading x can be given as a percentage of the full-load apparent power or load current

$$x = \frac{I_2'}{{I_2'}^{F.L}} = \frac{S_{out}}{S_{out}^{F.L}}$$

2.5 Maximum Efficiency

From the efficiency equation, it can be concluded that, the efficiency of a transformer is zero at no load. It increases with the increase in the load and rises to a maximum value. Any further increase in the load actually forces the efficiency of a transformer to drop off. Therefore, the efficiency of a transformer is maximized for a definite load. We now proceed to determine the criterion for the maximum efficiency of a transformer.

Normally, load voltage remains fixed. Therefore, efficiency depends on the load current I_2' and load power factor $\cos\theta_2$. In order to derive the conditions for maximum efficiency of the transformer at a constant load power factor,

$$\frac{d\eta}{dI_2'} = 0$$

$$\frac{(V_2'I_2'\cos\theta_2 + P_c + I_2'^2R_{e1})V_2'\cos\theta_2 - V_2'I_2'\cos\theta_2\,(V_2'\cos\theta_2 + 2I_2'R_{e1})}{(V_2'I_2'\cos\theta_2 + P_c + I_2'^2R_{e1})^2} = 0$$

$$(V_2'I_2'\cos\theta_2 + P_c + I_2'^2R_{e1})V_2'\cos\theta_2 - V_2'I_2'\cos\theta_2\,(V_2'\cos\theta_2 + 2I_2'R_{e1}) = 0$$

$$V_2'I_2'\cos\theta_2 + P_c + I_2'^2R_{e1} - I_2'(V_2'\cos\theta_2 + 2I_2'R_{e1}) = 0$$

$$V_2'I_2'\cos\theta_2 + P_c + I_2'^2R_{e1} - V_2'I_2'\cos\theta_2 - 2I_2'^2R_{e1} = 0$$

$$P_c = I_2'^2R_{e1}$$

Then, the condition for maximum efficiency at constant load terminal voltage and power factor is

$$P_{cu} = P_c$$

For different loading conditions

$$P_c = x^2 P_{cu}^{\ F.L}$$

Therefore, the per-unit loading for maximum efficiency at constant load terminal voltage and power factor is

$$x|_{\eta\max} = \sqrt{\frac{P_c}{P_{cu}^{\ F.L}}}$$

Then, the maximum efficiency at this condition can be determined as follows:

$$\%\,\eta = \frac{x|_{\eta\max}\cdot S_{out}^{\ F.L}\cos\varphi}{x|_{\eta\max}\cdot S_{out}^{\ F.L}\cos\varphi + 2P_c}\times 100$$

The above condition states that the efficiency of a transformer at constant load terminal voltage and power factor is maximum when the copper loss is equal to the core (magnetic) loss. In other words, a transformer operates at its maximum efficiency when the copper-loss curve intersects the core-loss curve, as depicted in Fig. 2.22.

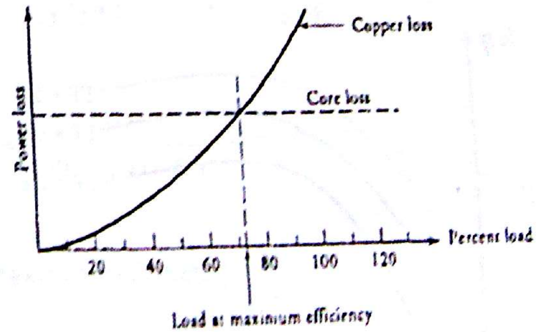


Fig. 2.22. Losses in a real transformer and condition for max. efficiency.

On the other hand, for constant values of the load terminal voltage V_2 and load current I_2, the maximum efficiency occurs when

$$\frac{d\eta}{d\theta_2} = 0$$

$$\frac{-(V_2'I_2'\cos\theta_2 + P_c + I_2'^2 R_{e1})V_2'I_2'\sin\theta_2 + (V_2'I_2')^2\cos\theta_2\sin\theta_2}{(V_2'I_2'\cos\theta_2 + P_c + I_2'^2 R_{e1})^2} = 0$$

$$-(V_2'I_2'\cos\theta_2 + P_c + I_2'^2 R_{e1})V_2'I_2'\sin\theta_2 + (V_2'I_2')^2\cos\theta_2\sin\theta_2 = 0$$

$$V_2'I_2'\sin\theta_2\left(-V_2'I_2'\cos\theta_2 - P_c - I_2'^2 R_{e1} + V_2'I_2'\cos\theta_2\right) = 0$$

$$-V_2'I_2'\sin\theta_2\left(P_c + I_2'^2 R_{e1}\right) = 0$$

$$V_2'I_2'\sin\theta_2 = 0$$

Therefore, the condition for maximum efficiency in this case is

$$\sin\theta_2 = 0 \xrightarrow{yields} \theta_2 = 0$$

$$\cos\theta_2 = 1$$

Therefore, maximum efficiency in a transformer occurs when the load *p.f* is unity and load current is such that copper loss equals core-loss. Figure 2.23 shows the variation of efficiency with load current and load power factor.

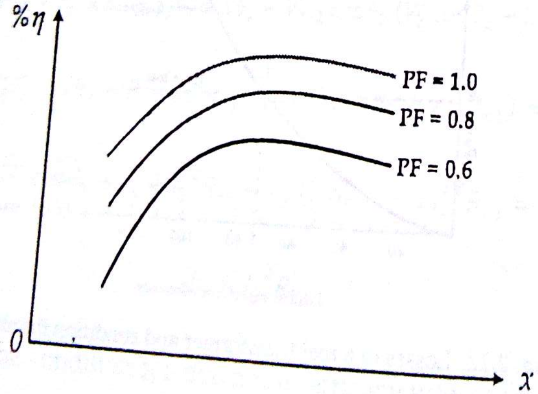


Fig. 2.23. Efficiency of a transformer

EXAMPLE 2.4

For the transformer in Example 2.1 , determine

a) Efficiency at 75% rated output and 0.6 PF.

b) Power output at maximum efficiency and the value of maximum efficiency. At what percent of full load does this maximum efficiency occur?

Solution

(a) $P_{out} = V_2 I_2 \cos\varphi_2$.

$$= 0.75 * 10000 * 0.6 = 4500\,W$$

$$P_c = 100W\,,$$

$$P_{cu} = I_1^2 R_{eq1}$$

$$= (0.75 * 4.55)^2 * 10.4 = 121W$$

$$\eta = \frac{4500}{4500 + 100 + 121} * 100 = 95.32\%$$

(b) At maximum efficiency

$$P_{core} = P_{cu} \quad and \quad PF = \cos\varphi_2 = 1$$

Now, $P_{core} = 100W = I_2^2 R_{eq2} = P_{cu}$

0·
1

$$\text{Then, } I_2 = \left(\frac{100}{0.104}\right)^{1/2} = 31\,A$$

$$P_{out}|_{\eta_{max}} = V_2 I_2 \cos\varphi_2 = 220 * 31 * 1 = 6820\,W$$

$$\eta_{max} = \frac{P_{out}|_{\eta_{max}}}{P_{out}|_{\eta_{max}} + P_c + P_{cu}} = \frac{6820}{6820 + 100 + 100} * 100$$

$=97.15\%$

Output kVA=6.82 and Rated kVA=10

Then, η_{max} occurs at 68.2% full load.

Another Method

From *Example 2.1* $P_{cu,FL} = 215W$

$$\text{Then } X = \sqrt{\left(\frac{P_c}{P_{cu,FL}}\right)} = \sqrt{\left(\frac{100}{215}\right)} = 0.68$$

2.6 All-Day Efficiency, η_{AD}

In the power plant, the transformer usually operates near its full load and is taken out of circuit when it is not required. Such transformers are called *power transformers*. It has been designed for maximum efficiency occurring near the rated output.

In the distribution system, a transformer connected to the utility that supplies power to your house and the locality is called a *distribution transformer*. Such transformers are connected to the power system 24 hours a day and operate well below the rated power output for most of the time. It is therefore desirable to design a distribution transformer for maximum efficiency occurring at the average output power.

A figure of merit that will be more appropriate to represent the efficiency performance of a distribution transformer is the "all-day" or "energy" efficiency of the transformer " η_{AD}". This is defined as follows:

$$\%\,\eta_{AD} = \frac{E_{out}}{E_{in}} \times 100$$

$$\%\,\eta_{AD} = \frac{E_{out}}{E_{out} + E_{loss}} \times 100$$

where E_{out} is the Energy output over 24 hr and it can be calculated as $E_{out} = \sum P_{out} \cdot t$

E_{in} is the Energy input over 24 hr

E_{loss} is the Energy losses over 24 hr and it can be calculated $E_{loss} = 24 \times P_c + \sum x^2 P_{cu} \cdot t$

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EXAMPLE 2.5

A 50 kVA, 2400/240 V transformer has a core loss P_c = 200 W at rated voltage and a copper loss Pcu = 500 W at full load. It has the following load cycle.

%Load	0.0%	50%	75%	100%	110%
Power Factor		1	0.8 lag	0.9 lag	1
Hours	6	6	6	3	3

Determine the all-day efficiency of the transformer.

Solution

Energy output 24 hours is

0.5*50*6+0.75*50*0.8*6+1*50*0.9*3+1.1*50*1*3=630 kWh

Energy losses over 24 hours:

Core loss =0.2*24=4.8 kWh

Copper losses (kWh) =

$$0.5^2 * 0.5 * 6 + 0.75^2 * 0.5 * 6 + 1^2 * 0.5 * 3 + 1.1^2 * 0.5 * 3 = 5.76$$

Total energy loss=4.8+5.76=10.56 kWh

Then, $\eta_{AD} = \dfrac{630}{630 + 10.56} * 100 = 98.35\%$

2.7 Voltage Regulation

A constant load terminal voltage is one of the most important load requirements. Because a real transformer has series impedances, the output voltage of a transformer varies with the load even if the input voltage remains constant. To conveniently compare transformers in this respect, it is customary to define a quantity called *Voltage Regulation* (*VR*). Usually it is a good practice to have as small a voltage regulation as possible. For an ideal transformer, *VR* = 0.

The <u>*full-load voltage regulation*</u> is defined as the net change in the secondary winding voltage from no-load to full-load for the same primary winding voltage. As a percent, it may be written as

$$\% VR = \frac{|V_{2NL}| - |V_{2FL}|}{|V_{2FL}|} \times 100$$

where V_{2NL} and V_{2FL} are the rms values of no-load and full-load voltages at the secondary terminals. The no-load and full-load voltages can be calculated by using equivalent circuits referred to either primary or secondary.

Let us consider the approximately equivalent circuit referred to the primary side, shown in Fig. 2.14. The voltage regulation can be determined as

$$\% VR = \frac{|V_{2NL}'| - |V_{2FL}'|}{|V_{2FL}'|} \times 100$$

This yield:

$$\% VR = \frac{|V_1| - |V_2'|_{rated}}{|V_2'|_{rated}} \times 100$$

10·

where

$$V_1 = V_2' + I_2'(R_{e1} + jX_{e1})$$

$$V_1 = V_2'\angle 0° + I_2'\angle\theta_2° \times Z_{e1}\angle\theta_{e1}°$$

Therefore, the voltage regulation depends on the load current, I_2' power factor " $\cos\theta_2$ ". Figure 2.24 shows voltage regulation of a real transformer versus load current at a different power factor.

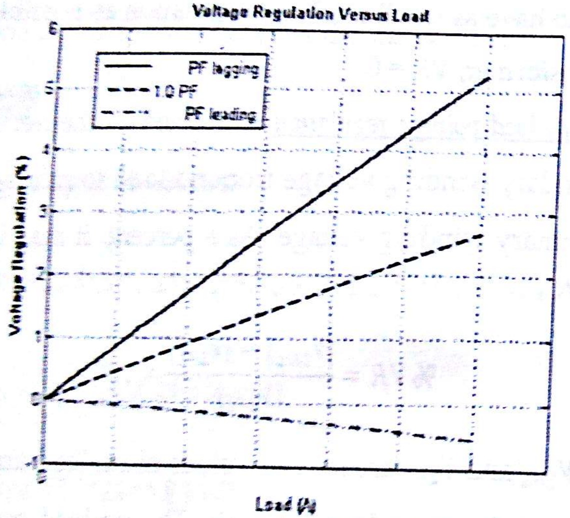

Fig. 2.24. The voltage regulation versus load current at different power factor

It can be noted from Fig. 2.24 that, the voltage regulation is positive at any load for both unity and lagging power factor. At unity power factor, the voltage regulation is a smaller number than it was with a lagging power factor. While, at leading power factor, the voltage regulation can actually has a negative voltage regulation (see Fig. 2-24).

In order to derive the conditions for maximum voltage regulation, consider the phasor diagram of the approximately equivalent circuit at lagging power factor as in Fig. 2.25.

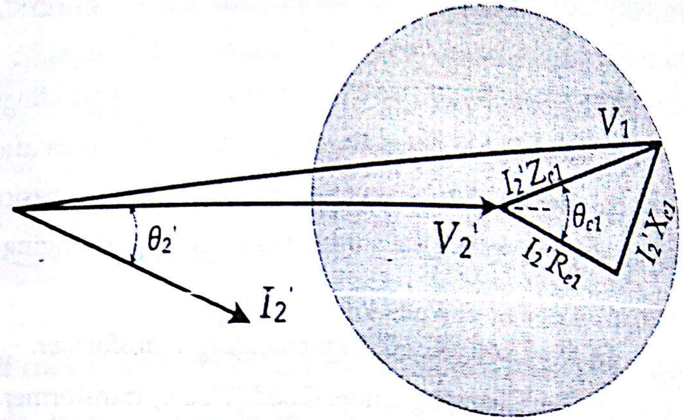

Fig. 2.25. The phasor diagram of an approximately equivalent circuit of a transformer referred to the primary side of the lagging power actor

The locus of V_1 is a circle of radius $I_2'Z_{e1}$. The magnitude of V_1 <u>*will be maximized*</u> if the phasor $I_2'Z_{e1}$ is in-phase with V_2'. This means:

$$\theta_2 = -\theta_{e1}$$

Therefore, the maximum voltage regulation " VR_{max} " occurs if the power factor angle of the load θ_2 is the same as the transformer equivalent impedance angle θ_{e1} and the <u>load power factor is lagging</u>.

<u>***Exercise:***</u>

Prove that: $$\% VR_{max} = \frac{\lvert I_2'Z_{e1}\rvert}{\lvert V_2'\rvert_{rated}} \times 100$$

2.8 Transformer Taps and Voltage Regulator

In all previous sections, the turn's ratio of a transformer was considered as though it were completely fixed. However, in almost all real *distribution transformers*, this is not quite true.

Distribution transformer has a series of *taps* in the windings to permit small changes in the turn's ratio of the transformer after it has left the factory. These types of transformers are basically classified according to the mechanism of tap changing as follows:

(a) No-Load (NLTC) Tap-changing Transformer.

(b) Tap Changing Under-Load (TCUL) transformer.

2.8.1 No-Load Tap-Changing Transformer

The No-Load Tap Changer (NLTC) also called off-circuit tap changer and *De-Energized Tap Changer (DETC)*. It is used in low power transformer. Figure 2.26 shows the arrangement where a number of tappings have been provided on the secondary of a transformer. As the position of the tap is varied, the effective number of secondary turns is varied and hence the output voltage of the secondary can be changed. The same purposes can be accomplished with the taps in the primary winding.

The tap changing device is usually kept on the high voltage side so that the current flow through the device will be low. However, the *principal disadvantage* of this type is that it cannot be used for tap-changing on load. Suppose for a moment that

tapping is changed from position 1 to position 2 when the transformer is supplying load.

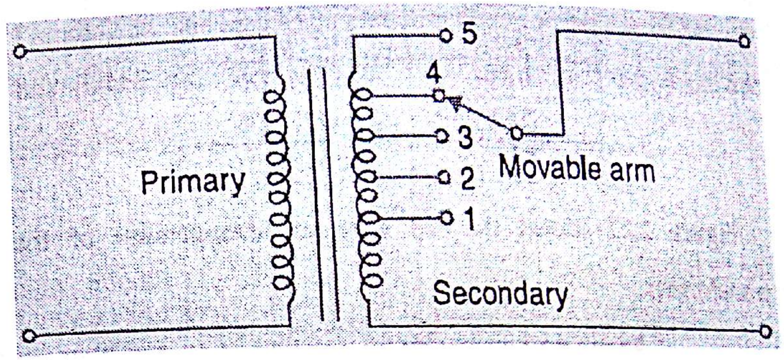

Fig. 2.26.Tap changing at no-load transformer.

If contact with stud 1 is broken before contact with stud 2 is made, there is break in the circuit and arcing results. On the other hand, if contact with stud 2 is made before contact with stud 1 is broken; the coils connected between these two tappings are short-circuited and carry damaging heavy currents. For this reason, this circuit arrangement cannot be used for tap-changing on load.

An Example of Transformer Tap-changing

Standard tap ranges for distribution transformers usually cover a tap range of 10% in four steps of 2.5%, or, in some ratings, two steps of 5%. For example, if a 100-kVA, 11,000/220-V distribution transformer has four 2.5% taps on its primary winding. What are the voltage ratios of this transformer at each tap setting? Also, draw the tap arrangement of the transformer?

- The five possible voltage ratings are

Taps	Rated Voltages
+5.0%	11,550/220
+2.5%	11,275/220
0.0%	11,000/220
-2.5%	10,725/220
-5.0%	10,450/220

Figure 2.27 shows the tap changer arrangement of the transformer.

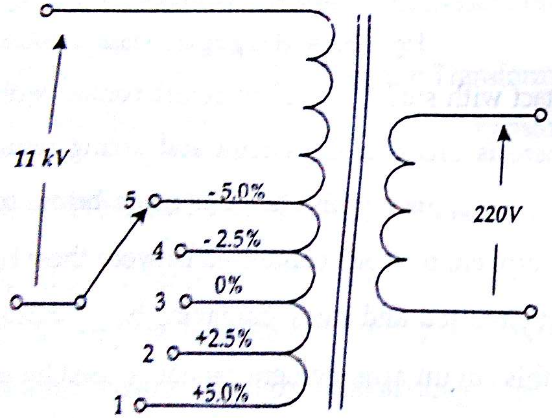

Fig. 2.27. Tap Changer arrangement of 11kV/220V Transformer

2.8.2 Tap-Changing Under Load Transformer

A Tap Changing under Load (TCUL) transformer is a transformer with the ability to change taps while power is connected to it. It contains a built-in voltage sensing circuit that automatically changes taps to keep the system voltage constant. Such special transformers are very common in modern power systems.

TCUL changers may be generally classified as:

1- Changing by impedance or reactor transition and

2- Changing using equal parallel windings.

Figure 2.28 shows the schematic diagram of the TCUL by impedance transition. While, Fig. 2.29 shows the schematic diagram of the TCUL equal parallel windings

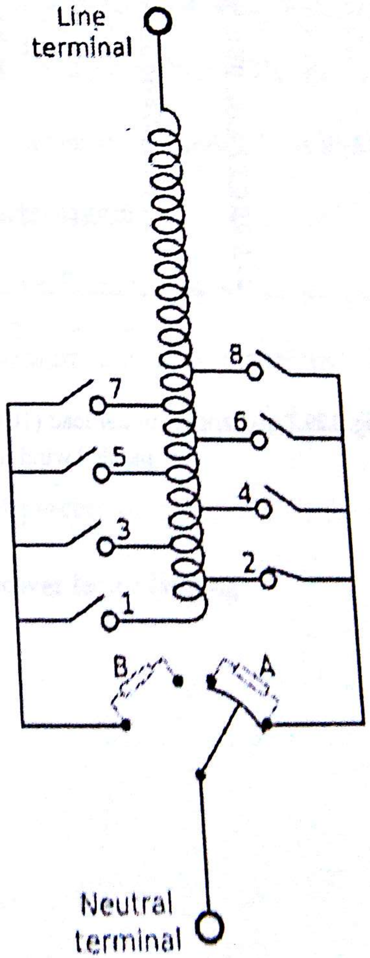

Fig. 2.28. Tap changing under load (TCUL) transformer by impedance or reactor transition and

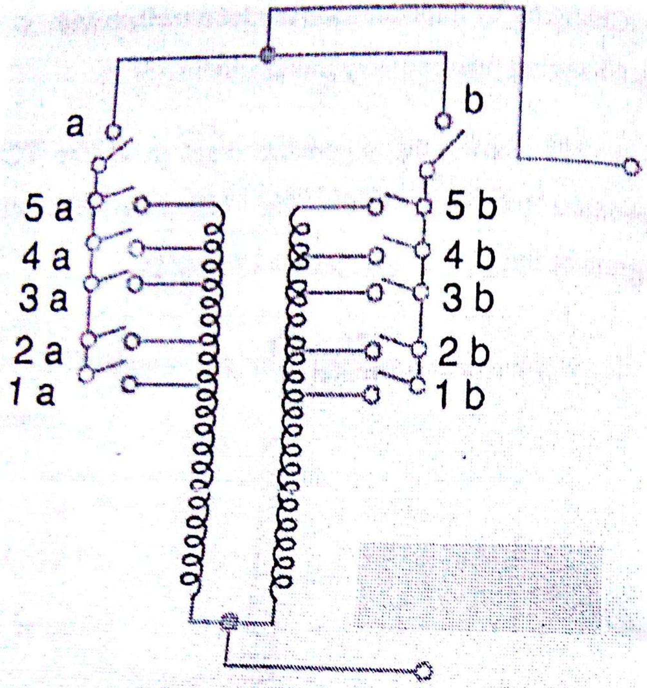

Fig. 2.29. Tap changing under load (TCUL) transformer using parallel windings

EXAMPLE 2.6 **Report**

A 6kVA, 250/500 V, transformer gave the following test results.

Short-circuit: 20 V ; 12 A, 100 W and Open-circuit test : 250 V, 1 A, 80 W

a) Determine the transformer equivalent circuit.

b) Calculate applied voltage, voltage regulation and efficiency when the output is 10 A at 500 volt and 0.8 power factor lagging.

c) Maximum efficiency, at what percent of full load does this maximum efficiency occur? (At 0.8 power factor lagging).

d) At what percent of full load does the effeciency is 95% at 0.8 power factor lagging.

2.9 Per-Unit System

Calculations using the actual values of parameters and variables are lengthy and time consuming. However, if the quantities are expressed in a per-unit (p.u) system, calculations are simplified. In addition, the p.u system represents another approach to solving circuits containing transformers, which eliminates the need for explicit voltage-level conversions (voltage, current and impedance transformations) at every transformer in the system.

The p.u quantity is defined as follows:

$$\text{p.u value} = \frac{\text{actual value}}{\text{base value}}$$

where for any electric system has four base quantities: voltage, current, apparent power, and impedance. If we select base values of any two of them, the base values of the remaining two can be calculated. If S_b is the apparent base power and V_b is the base voltage, then the base current I_b and base impedance Z_b are:

$$I_b = \frac{S_b}{V_b}$$

$$Z_b = \frac{V_b}{I_b} = \frac{{V_b}^2}{S_b}$$

Since the power rating of a transformer is the same on both sides, we can use it as one of the base quantities. However, we have to select two base voltages one for the primary side and the other for the secondary side. Therefore, there are two base quantities of current I_{b1}, I_{b2} and two base impedances for both sides Z_{b1}, Z_{b2}.

Exercise:

Prove that, the per-unit transformer voltage, current and impedance is the same referred to either side of the transformer.

2.9.1 Transformer Equivalent Circuit in P.U

The equivalent circuit of a transformer referred to the primary side is shown in Fig. 2.30.

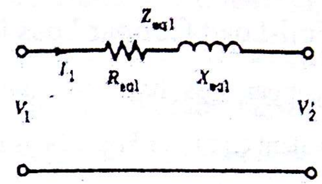

Fig. 2.30. Transformer equivalent circuit in actual values.

By applying KVL, the primary voltage equation in terms of actual values is

$$V_1 = V_2' + I_1 Z_{e1}$$

The equation in per-unit system can be obtained by dividing the above equation by the base value of the primary voltage V_{b1}.

$$\frac{V_1}{V_{b1}} = \frac{V_2'}{V_{b1}} + \frac{I_1 Z_{e1}}{V_{b1}}$$

$$V_{1,p.u} = \frac{aV_2}{aV_{b2}} + \frac{I_1 Z_{e1}}{I_{b1} Z_{b1}}$$

$$V_{1,p.u} = V_{2,p.u} + I_{1,p.u} Z_{e1,p.u}$$

Therefore, the equivalent circuit in per-unit system can be represented as in Fig. 2.31. It has been shown that the voltages, currents, and impedances in per-unit representation have the same values whether they are referred to primary or secondary.

Note that the values of $V_{1,pu}$ and $V_{2,pu}$ are generally close to 1 p.u, and this makes the analysis somewhat easier.

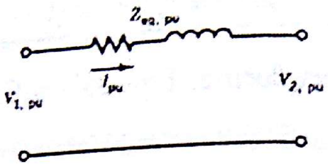

Fig. 2.31.Transformer equivalent circuit in per-unit values.

2.9.2 Full-Load Copper Loss in P.U

The full-load copper loss has been determined from the approximate equivalent circuit of Fig. 2.14 as follows:

$$P_{cu}^{F.L} = I_{1,F.L}^2 \cdot R_{e1}$$

Therefore, it can be calculated in per-unit system based on the volt-ampere rating of the transformer as follows:

$$P_{cu}^{F.L}\Big|_{p.u} = \frac{I_{1,F.L}^2 \cdot R_{e1}}{S_b}$$

$$= \frac{I_{1,F.L}^2 \cdot R_{e1}}{V_{b1} \cdot I_{b1}}$$

Since

$$I_{b1} = I_{1,F.L}$$

$$Z_{b1} = \frac{V_{b1}}{I_{b1}}$$

Therefore

$$P_{cu}^{F.L}\Big|_{p.u} = R_{e1,p.u}$$

Hence, the full-load copper loss in per-unit system equals the transformer resistance expressed in per-unit. The per-unit value of the resistance is therefore more useful than its actual value in determining the performance of a transformer.

2.10 Parallel Operations of Single-Phase Transformers (1)

In order to supply a load in excess of the rating of an existing transformer, a second transformer may be connected in parallel with it as shown in Fig. 2.32. It is seen that primary windings are connected to the supply bus bars and secondary windings are connected to the load bus-bars. In connecting two or more than two transformers in parallel, it is essential that their terminals of similar polarities are joined to the same bus-bars as in Fig. 2.32. If this is not done, the two e.m.fs. induced in the secondaries which are paralleled with incorrect polarities, will act together in the local secondary circuit even when supplying no load and will hence produce the equivalent of a dead short-circuit.

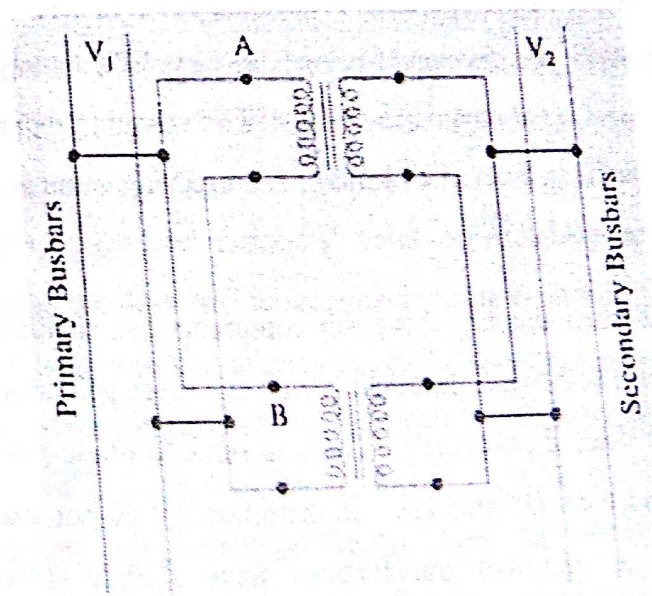

Fig. 2.32.Connection of two transformers in parallel

There are certain definite conditions, which must be satisfied in order to avoid any local circulating currents and to ensure that the transformers share the common load in proportion to their kVA ratings. The conditions are:

1) Primary windings of the transformers should be suitable for the supply system voltage and frequency.
2) The transformers should be properly connected with regard to polarity.
3) The voltage ratings of both primaries and secondaries should be identical. In other words, the transformers should have the same turn ratio i.e. transformation ratio.
4) The percentage impedances should be equal in magnitude and have the same X/R ratio in order to avoid circulating currents and operation at different power factors.
5) With transformers having different kVA ratings, the equivalent impedances should be inversely proportional to the individual kVA rating if circulating currents are to be avoided.

Of these conditions, (1) is easily comprehended; condition (2) is essential (otherwise paralleling with incorrect polarities will result in dead short-circuit). There is some lattitude possible with conditions (3) and (4). If condition (3) is not exactly satisfied i.e. the two transformers have slightly different transformation or voltage ratios, even then parallel operation is possible. However, due to inequality of induced emfs in secondaries, there will be even on no-load, some circulating current between them (and therefore between the primary windings also) when secondary terminals are connected in parallel. When secondaries are loaded, this localized circulating current will tend to produce unequal loading condition. Hence, it may be impossible to take full kVA output from the parallel-connected group without one of the transformers becoming over-heated.

If condition (4) is not exactly satisfied, i.e. impedance triangles are not identical in shape and size, parallel operation will still be possible, but the power factors at which the two transformers operate will be different from the power factor of the common load. Therefore, in this case, the two transformers will not share the load in proportion to their kVA ratings.

It should be noted that the impedances of two transformers might differ in magnitude and in quality (i.e. ratio of equivalent resistance to reactance). It is worthwhile to distinguish between the percentage and numerical value of an impedance. For example, consider two transformers having ratings in the ratio 1:2. It is obvious that to carry double the current, the latter must have half the impedance of the former for the same regulation. For parallel operation, the regulation must be the same, this condition being enforced by the very fact of their being connected in parallel. It means that the currents carried by the two transformers are proportional to their ratings provided their numerical impedances are inversely proportional to these ratings

and their percentage impedances are identical. If the ratio of percentage resistance to reactance is different, then this will result in divergence of phase angle of the two currents, with the result that one transformer will be operating with a higher and the other with a lower power factor than that of the combined load.

(a) Case 1: Ideal Case

We will first consider the ideal case of two transformers having the same voltage ratio and having impedance voltage triangles identical in size and shape. Let E be the no-load secondary voltage of each transformer and V_2 the terminal voltage; I_A and I_B the currents supplied by them and I-the total current, lagging behind V_2 by an angle φ as shown in Fig. 2.33.

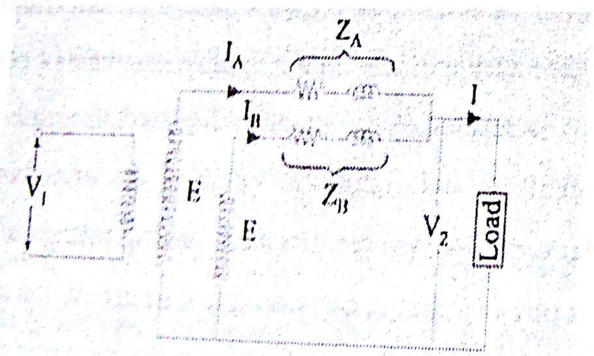

Fig. 2.33. Equivalent-circuit of case-1 and case-2

In Fig. 2.34 a single triangle, ABC represents the identical impedance voltage triangles of both the transformers. The currents I_A and I_B of the individual transformers are in-phase

with the load current I and are inversely proportional to the respective impedances. Following relations are obvious.

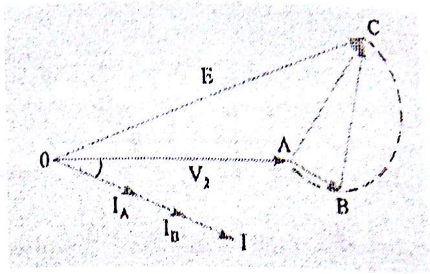

Fig. 2.34. Phasor -diagram of case-1

$$I = I_A + I_B$$
$$V_2 = E - I_A Z_A$$
$$V_2 = E - I_B Z_B$$
$$V_2 = E - I Z_{AB}$$

Also

$$I_A Z_A = I_B Z_B$$
$$\frac{I_A}{I_B} = \frac{Z_B}{Z_A}$$

$$\boxed{\therefore\ I_A = I\frac{Z_B}{Z_A + Z_B} \quad \& \quad I_B = I\frac{Z_A}{Z_A + Z_B}}$$

(b) Case 2: Equal Voltage Ratios

Let us assume that no-load voltages of both secondaries is the same i.e. $E_A = E_B = E$ and that the two voltages are coincident i.e. there is no-phase difference between E_A and E_B, which would be true if the magnetizing currents of the two transformers are not much different from each other. Under these conditions, both primaries and secondaries of the two transformers can be

connected in parallel and there will circulate no current between them on on-load. The phasor diagram is shown in Fig. 2.35.

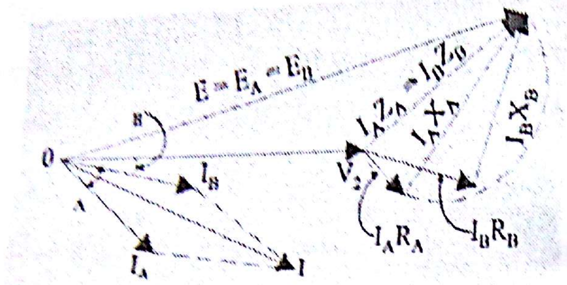

Fig. 2.35.Phasor –diagram of case-2

$$\therefore I_A = I\frac{Z_B}{Z_A + Z_B} \quad \& \quad I_B = I\frac{Z_A}{Z_A + Z_B}$$

2.11 Transient Inrush Current [2]

In normal steady-state operation, the magnetization current of a transformer is usually very low – less than 5 percent of rated conditions. However, now when a transformer is connected to the power system, a large inrush current will flow in the transformer during the transient period. This current may be as high as 10 to 20 times the rated current. Knowledge of this large inrush current is important in determining the maximum mechanical stresses that could occur in the transformer windings and in designing the protective system for the transformer. The magnitude of the inrush current depends on the instant of the voltage wave at which the transformer is connected to the power supply. Consider a transformer whose core is initially un-magnetized. The transformer primary winding is now connected to a supply voltage

$$v = \sqrt{2}\, V \sin \omega t$$

If we neglect the core losses and the primary-winding resistance,

[2] Electric Machinery Fundamentals, by: Stephen J. Chapman, pp. 139-140.

$$v = N\frac{d\Phi}{dt}$$

$$\Phi = \frac{1}{N}\int v\,dt$$

Consider two cases, as follows:

Case 1. The transformer is connected when the voltage is maximum. The voltage and flux variations for this situation are shown in Fig. 2.36. Note that there is no transient in flux, and that the time variation of flux is

$$\Phi = \Phi_{max}\sin(\omega t - 90^\circ) \qquad \text{for} \quad \omega t > 90^\circ$$

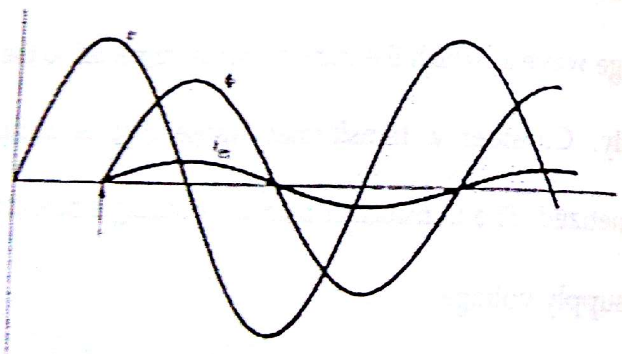

Fig. 2.36. Transformer Inrush Current (case-1)

where

$$\Phi_{max} = \frac{\sqrt{2}V}{\omega N}$$

The magnetizing current can be found from the B-H curve of the transformer core and is also shown in Fig. 2.36. No inrush current will flow and the system is in steady state from the start.

Case 2. The transformer is connected when the voltage is zero. The time-variation of flux can be found as follows:

$$v = \sqrt{2}\,V\sin\omega t$$

$$v = N\frac{d\Phi}{dt}$$

$$\Phi = \frac{1}{N}\int v\,dt$$

$$\Phi = \frac{\sqrt{2}V}{N}\int_0^t \sin\omega t\,dt$$

$$= \frac{\sqrt{2}V}{\omega N}(1 - \cos\omega t)$$

$$= \Phi_{max} - \Phi_{max}\cos\omega t$$

The time variations of voltage, flux, and magnetizing current are shown in Fig. 2.37. The peak flux has doubled and the corresponding peak magnetizing current is very large because of core saturation.

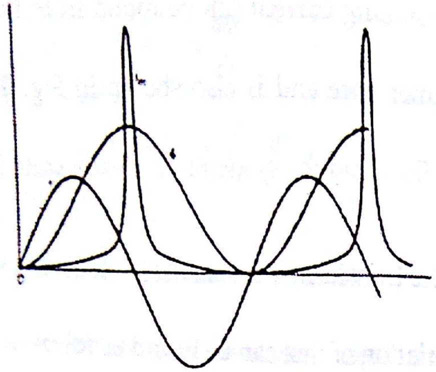

Fig. 2.37. Transformer Inrush Current (case-2)

In practice, because of winding resistance, the large inrush current will decay rapidly, as shown in Fig. 2.38. In a three-phase transformer, there is always an inrush current, because even if the voltage is a maximum for one phase at the instant the transformer is connected to the power supply, it is not maximum for the other phases.

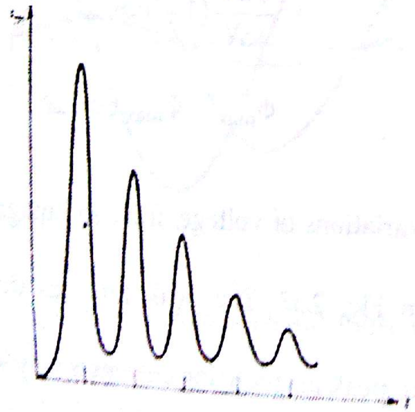

Fig. 2.38. Transformer Inrush Current if the effect of winding resistances is considered.

Review Questions (2)

- **Real Transformer**

22. What are (List and describe) the various losses occurring in a transformer?

23. Why does a practical transformer draw some current when its secondary winding is open?

24. What components compose the excitation current of a transformer? Distinguish between them. How are they modeled in the transformer equivalent circuit?

25. What is a leakage flux? How can the leakage flux be minimized? Is it possible to have no leakage flux? Why is it modeled in a transformer equivalent circuit as a series inductor?

26. Explain voltage regulation. When a transformer operates at half load, the secondary winding voltage is 250 V. At no load, it is 270 V. What is the voltage regulation?

27. The no-load voltage is lower than the full-load voltage on the secondary side of a transformer. Under what conditions can this happen?

28. Why does the power factor of a load affect the voltage regulation of a transformer?

29. What are transformer taps? Why are they used?

30. Why is an open circuit test conducted at the rated voltage?

31. Outline carefully the procedure for performing the open-circuit test.

32. What is the advantage of performing open-circuit test on the low - voltage side?

33. Why is a short-circuit test performed at the rated current?

34. Outline carefully the procedure for performing the short-circuit test.

35. Why do we prefer to perform the short-circuit test on the high-voltage side?

36. Explain why the load does not affect the core loss.

37. Explain why the copper loss varies with the load.

38. What is the criterion for maximum efficiency of a transformer?

39. Deduce the criterion for maximum efficiency of a transformer.

40. In a distribution transformer, the maximum efficiency occurs at 50% of the load. What does this mean? If the operating frequency is increased, what happens to the load current at maximum efficiency?

41. How does a change in frequency affect the operation of a given transformer (operation parameters are iron & copper losses, efficiency and regulation)?

42. When a transformer is connected to a 1000-V, 50-Hz supply the core loss is 1000 W, of which 650 is hysteresis and 350 is eddy current loss. If the applied voltage is raised to 2,000 V and the frequency to 100 Hz, find the new core losses.

43. In a power loss test on a 10 kg specimen of sheet steel laminations, the maximum flux density and waveform factor are maintained constant and the following results were obtained:

Frequency (Hz)	25	40	50	60	80
Total loss (watt)	18.5	36	50	66	104

Calculate the eddy current loss per kg at a frequency of 50 Hz.

44. Using the apparent power and the terminal voltages as the base quantities in a 44-kVA, 1100/250-V transformer, determine the other base quantities.

45. What is the a-ratio on a per-unit basis?

46. Draw and explain the phasor diagram of a transformer on load at a lagging power factor.

47. Write short notes on parallel operation of transformers.

48. What are the conditions for satisfying parallel operation of single-phase transformer? Deduce an expression for the load shared by the two transformers in parallel when the transformers have an equal voltage ratio.

49. Write a short note about the transformer inrush current.

50. What happens to a transformer when it is first connected to a power line? Can anything be done to mitigate this problem.

51. A 1-phase, 100 kVA, 1000/100 V transformer gave the following test results: open-circuit test 100 V, 6.0 A, 400 W short-circuit test 50 V, 100 A, 1800 W

a) Determine the rated voltage and rated current for the HV and LV sides.

b) Derive an approximate equivalent circuit referred to the HV side.

c) Determine the regulation at full load, 0.6 PF leading.

d) Draw the phasor diagram for condition (c).

52. A 1-phase, 25 kVA, 220/440 V, 60 Hz transformer gave the following test results.

Open circuit test : 220 V, 9.5 A, 650 W

Short-circuit test: 37.5 V, 55 A, 950 W

(a) Derive the approximate equivalent circuit in per-unit values. (b) Determine the voltage regulation at full load, 0.8 PF lagging. (c) Draw the phasor diagram for condition (b).

53. A 1-phase, 10 kVA, 2400/ 120 V, 60 Hz transformer has the following equivalent circuit parameters: $Zeq_1 = 5 + j25\ \Omega$, $Rc_1 = 64\ k\Omega$ and $Xm_1 = 9.6\ k\Omega$ Standard no-load and short-circuit tests are performed on this transformer. Determine the following:

No-load test results: V_{oc}, I_{oc}, P_{oc}

Short-circuit test results: V_{sc}, I_{sc}, P_{sc}

54. A single-phase, 250 kVA, 11 kV/2.2 kV, 60 Hz transformer has the following parameters. R_{HV}= 1.3 Ω X_{HV}=4.5Ω, R_{LV} = 0.05 Ω, X_{LV} = 0.16, R_{c2}= 2.4 kΩ X_{m2} = 0.8 kΩ

a) Draw the approximate equivalent circuit (i.e., magnetizing branch, with R_{c1} and X_m connected to the supply terminals) referred to the HV side and show the parameter values.

b) Determine the no load current in amperes (HV side) as well as in per unit.

c) If the low-voltage winding terminals are shorted, determine:

i. The supply voltage required to pass rated current through the shorted winding.

ii. The losses in the transformer.

iii. The HV winding of the transformer is connected to the 11 kV supply and a load, $Z_L = 15\angle -90^\circ\ \Omega$ is connected to the low voltage winding. Determine: the load voltage. and voltage regulation.

55. A 1-phase, 10 kVA, 2400/240 V, 60 Hz distribution transformer has the following characteristics: Core-loss at full-voltage = 100 W Copper-loss at half-load = 60 W.

a) Determine the efficiency of the transformer when it delivers full load at 0.8 power factor lagging.

b) Determine the per unit rating at which the transformer efficiency is a maximum.

c) Determine this efficiency if the load p.f. is 0.9.

d) The transformer has the following load cycle: No-load for 6 hours 70% full load for 10 hours at 0.8 PF 90% full load for 8 hours at 0.9 PF. Determine the all-day efficiency of the transformer.

Solved Problems

Q1) A 250/500 V, transformer gave the following test results

Short-circuit test : with low-voltage winding shorted.

short-circuited 20 V ; 12 A, 100 W

Open-circuit test : 250 V, 1 A, 80 W on low-voltage side.

Determine the circuit constants, insert these on the equivalent circuit diagram and calculate applied voltage, voltage regulation and efficiency when the output is 5 A at 500 volt and 0.8 power factor lagging.

Solution

Open circuit test

$$\cos\varphi_o = \frac{P_{oc}}{V_{oc} I_{oc}} = \frac{80}{250 * 1} = 0.32$$

$$I_{c1} = I_o \cos\varphi_o = 1 * 0.32 = 0.32 A$$

$$I_{m1} = \sqrt{I_o^2 - I_c^2} = \sqrt{1^2 - 0.32^2} = 0.95 A$$

$$R_{c1} = \frac{V_{1oc}}{I_c} = \frac{250}{0.32} = 781.3\ \Omega$$

$$X_{m1} = \frac{V_{1oc}}{I_m} = \frac{250}{0.95} = 263.8\ \Omega$$

Short circuit test

As the primary is short-circuited, all values refer to secondary winding. So we can obtain R_{eq2} and X_{eq2} and then refer them to primary to get R_{eq1} and X_{eq1} as explained before in Example or we can modify the short circuit data to the primary and then we can calculate R_{eq1} and X_{eq1} directly. Here will use the two method to compare the results.

First method

$$R_{eq2} = \frac{P_{sc}}{I_{2sc}^2} = \frac{100}{12^2} = 0.694\ \Omega$$

$$Z_{eq2} = \frac{V_{sc}}{I_{2sc}} = \frac{20}{12} = 1.667\ \Omega$$

Then, $X_{eq2} = \sqrt{Z_{eq2}^2 - R_{eq2}^2} = \sqrt{1.667^2 - 0.694^2} = 1.518\ \Omega$

As R_c and X_m refer to primary, hence we will transfer these values (R_{eq2}, X_{eq2}, and Z_{eq2}) to primary with the help of transformation ratio.

Then

$$R_{eq1} = a^2 * R_{eq2} = 0.5^2 * 0.694 = 0.174\ \Omega$$

$$X_{eq1} = a^2 * X_{eq2} = 0.5^2 * 1.518 = 0.38\Omega$$

$$Z_{eq1} = a^2 * Z_{eq2} = 0.5^2 * 1.667 = 0.417\Omega$$

Second method

Short-circuited results refeard to secondery are 20 V, 12 A, 100 W Then, Short-circuited results refeard to primary are 10 V, 24 A, 100 W

Then $R_{eq1} = \frac{P_{sc}}{I_{1sc}^2} = \frac{100}{24^2} = 0.174\ \Omega$

$Z_{eq1} = \frac{V_{1sc}}{I_{1sc}} = \frac{10}{24} = 0.417\ \Omega$

Then, $X_{eq1} = \sqrt{Z_{eq1}^2 - R_{eq1}^2} = \sqrt{0417^2 - 0.174^2} = 0.38\ \Omega$

Applied voltage

$$V_1\ \underline{\delta^o} = V_2'\ \underline{0^o} + I_2'\ \underline{\varphi^o} * Z_{eq1}$$

Then, $V_1\ \underline{\delta^o} = 250\ \underline{0^o} + 10\ \underline{-\cos^{-1} 0.8} * (0.174 + j0.38)$

$$V_1\ \underline{\delta^o} = 250\ \underline{0^o} + 10\ \underline{-36.24^o} * 0.418\ \underline{65.4^o}$$

$$V_1\ \underline{\delta^o} = 250\ \underline{0^o} + 4.18\ \underline{29.16^o}$$

$$V_1\ \underline{\delta^o} = 250\ \underline{0^o} + 3.65 + j2.04 = 253.65 + j2.04$$

$$= 253.7\ \underline{0.47^o}\ V$$

Voltage regulation

$$\%reg = \frac{(V_1) - (V_2')_{load}}{(V_2')_{load}} * 100$$

$$(V_2')_{load} = 250\ \underline{0^0}$$

$$\%reg = \frac{253.7 - 250}{250} * 100 = 1.48\%$$

Effeciency

$$\eta = \frac{V_2' * I_2' * \cos\varphi}{V_2' * I_2' * \cos\varphi + P_{cu} + P_{iron}} * 100$$

$$\eta = \frac{250 * 10 * 0.8}{250 * 10 * 0.8 + 10^2 * 0.174 + 80} * 100 = 95.356\%$$

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Q2) A 1ϕ, 10 kVA, 2400/240 V, 60 Hz distribution transformer has the following characteristics: Core loss at full voltage =100 W and Copper loss at half load =60 W (a) Determine the efficiency of the transformer when it delivers full load at 0.8 power factor lagging. (b) Determine the rating at which the transformer efficiency is a maximum. Determine the efficiency if the load p.f. is 0.9. (c) The transformer has the following load cycle: No load for 6 hours, 70% full load for 10 hours at 0.8 PF and 90% full load for 8 hours at 0.9 PF

Solution:

(a) $P_{out} = 10 * 0.8 = 8\ kW$

$$P_{core} = 100\ W,\quad P_{cu,FL} = 60 * 2^2 = 240W$$

$$\eta = \frac{8000}{8000 + 100 + 240} * 100 = 95.92\%$$

(b) $x = \sqrt{\frac{100}{240}} = 0.6455$

$$\eta_{max} = \frac{10 * 10^3 * 0.6455 * 0.9}{(10^4 * 0.6455 * 0.9) + 100 + 100} = 96.67\%$$

Output energy in 24 h...

$$E_{24hrs} = 0 + 10 * 0.7 * 0.8 * 10 + 10 * 0.9 * 0.9 * 8 = 120.8kWh$$

Energy losses in the core in 24 hours is

$$E_{core} = 100 * 24 * 10^{-3} = 2.4\,kWh$$

Energy losses in the cupper in 24 hours is

$$E_{cu} = \left(240 * 0.7^2 * 10 + 240 * 0.9^2 * 8\right) * 10^{-3} = 2.7312kWh$$

Then, $\eta_{all\ day} = \frac{120.8}{120.8 + 2.4 + 2.7312} * 100 = 95.93\%$

Q 3) Obtain the equivalent circuit of a 8kVA 200/400 V, 50 Hz, 1-phase transformer from the following test:-

O.C. test : 200 V, 0.8 A, 80W, S.C. test : 20 V, 20 A, 100 W

Calculate the secondary voltage when delivering 6 kW at 0.7 power factor lagging, the primary voltage being 200 V.

Solution

From open-circuit test:

$P_o = V_o I_o * \cos\varphi_o$

$\therefore \cos\varphi_o = \dfrac{P_o}{V_o I_o} = \dfrac{80}{200*0.8} = 0.5$

then $\varphi_o = \cos^{-1} 0.5 = 60^\circ$

then $I_{cl} = I_o \cos\varphi_o = 0.8*0.5 = 0.4A$

$I_{ml} = I_o \sin\varphi_o = 0.8*0.866 = 0.69282A$

then, $R_{cl} = \dfrac{V_{ol}}{I_{cl}} = \dfrac{200}{0.4} = 500\ \Omega$

and $X_{ml} = \dfrac{V_{ol}}{I_{cl}} = \dfrac{200}{0.69282} = 288.675\,\Omega$

From short circuit test:

It may be noted that in the test instruments have been placed in the secondary i.e. high voltage winding and the low voltage winding i.e. primary has been short-circuited.

Now,

$Z_{eq2} = \dfrac{V_{2sc}}{I_{2sc}} = \dfrac{20}{20} = 1\Omega$

Also, $P_{sc} = I_{2sc}^2\ R_{eq2}$

Then, $R_{eq2} = \dfrac{100}{20^2} = 0.25\,\Omega$

Then, $X_{eq2} = \sqrt{Z_{eq2}^2 - R_{eq2}^2} = \sqrt{1^2 - 0.25^2} = 0.968246\ \Omega$

Output current $I_2 = \dfrac{6000}{0.7*400} = 21.4286\,A$

Now, from the approximate equivalent circuit referred to secondary:

$V_2\angle 0^\circ = V_1'\angle\delta^\circ - I_2\angle\varphi^\circ * Z_{eq2}$

Then, $V_2\angle 0^\circ = 400\angle\delta^\circ - 21.4286\ \angle -45.573^\circ * (0.25 + j0.968246)$

$V_2\angle 0^\circ = 400\angle\delta^\circ - 21.43\ \angle 29.9495^\circ$

From the above equation, we have two unknown variables V_2 *and* δ° it need two equations to get both of them. The above equation is a complex one so we can get two equations out of it.

If we equate the real parts together and the equate the imaginary parts:

So from the Imaginary parts:

$$|V_2|\sin(0) = 400\sin(\delta^{o}) - 21.43 * \sin(29.9495^{o})$$

$$0 = 400 * \sin(\delta^{o}) - 10.6986$$

Then, $\delta^{o} = 1.533^{o}$

So from the Real parts:

$$|V_2|\cos(0) = 400 * \cos(1.533^{o}) - 21.43 * \cos(29.9495^{o})$$

Then, $|V_2| = 381.288\ V$

Chapter 3

Autotransformers

3.1 Introduction

The autotransformer is a special purpose transformer. It can be used when it is desirable to change the voltage levels by only a small amount. For example, it may be necessary to increase a voltage from 110 to 120 V or from 13.2 to 13.8 kV. These small rises may be made necessary by voltage drops that occur in power systems a long way from the generators.

It consists of a single winding unlike the two-winding transformer considered in chapters 1 and 2. The single winding is used as primary (or secondary) and the part of the winding is used as secondary (or primary). Therefore, the primary and secondary of an autotransformer are physically connected. Thus, the input and the output circuits are connected electrically, besides the magnetic coupling between them. *The direct electrical connection between the windings ensures that a part of the energy is transferred from the primary to the secondary by conduction. The magnetic coupling between the windings guarantees that some of the energy is also delivered by induction.*

It is common practice in power systems to use autotransformers whenever two voltages fairly close to each other in level need to be transformed, because the closer the two voltages are, the greater the autotransformer power advantage

becomes. They are also used as variable transformers, where the low-voltage tap moves up and down the winding. This is a very convenient way to get a variable ac voltage. Such a variable autotransformer is shown in Figure 3.1.

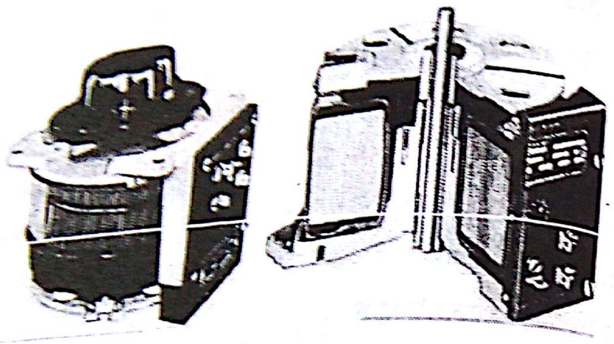


Fig. 3.1. Single-Phase Autotransformer

A tapping is provided on the winding to separate the primary and the secondary windings. So, in a single-phase autotransformer, there are three terminals. Figure 3.2 shows the connection diagram of a step-down autotransformer. The three-terminals are *a*, *c* and *b*. the primary winding is '*ab*', and the secondary winding is '*cb*'. It may be noted that '*cb*' winding is a common winding between primary and secondary sides.

The input voltage, which is applied to the primary winding, is V_H and the secondary output voltage is V_L. The primary current is I_H and the load current is I_L. If N_H is the number of turns of the primary winding and N_L or N_C is the number of turns of the secondary or common winding. As shown in Fig. 3.2,

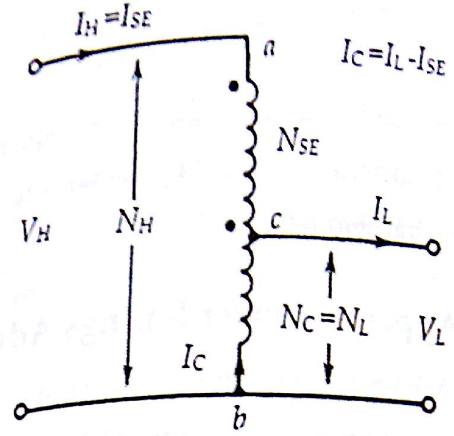


Fig. 3.2. Single-phase step-down autotransformer

$$N_H = N_{SE} + N_C$$

$$V_L = V_C$$

$$V_H = V_C + V_{SE}$$

$$I_H = I_{SE}$$

$$I_L = I_C + I_{SE}$$

3.2 Voltage and Current Relationships

Since, the basic principle of operation is the same as that of the two-winding transformer. Therefore,

$$\frac{V_H}{V_L} = \frac{N_H}{N_L} = \frac{N_{SE} + N_C}{N_C}$$

It can be found also from Fig. 3.1 that, for *mmf* balance,

$$N_{SE} I_H - N_C (I_L - I_H) = 0$$

$$N_{SE} I_H - N_C I_L + N_C I_H = 0$$

$$(N_{SE} + N_C) I_H = N_C I_L$$

$$\frac{I_H}{I_L} = \frac{N_C}{N_{SE} + N_C} = \frac{N_L}{N_H}$$

It can be concluded from the above equations that, the voltages and currents are related by the same turns ratio as in a two-winding transformer.

3.3 Apparent Power Ratings Advantage of Autotransformer

As mentioned before, the operation of an autotransformer is based on the conductive connection in addition to the electro-magnetic induction. Therefore, not all the power transferred from the primary to the secondary in the autotransformer goes through the windings. As a result, if a conventional two-winding transformer is reconnected as an autotransformer; it can handle much more power than it was originally rated for. To illustrate this idea, refer to Fig. 3.3.

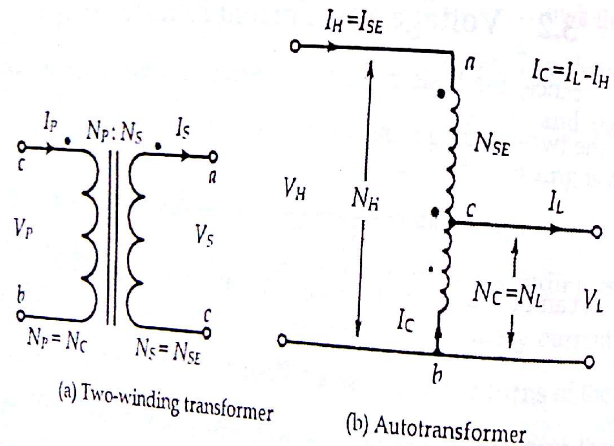


Fig. 3.3. Single-phase autotransformer

Notice that the input and output apparent power to the autotransformer is given by

$$S_{out} = V_L I_L$$

$$S_{in} = V_H I_H$$

It is easy to show, by using the voltage and current ratios that, the input apparent power is equal to the output apparent power:

$$S_{in} = S_{out} = S_{IO}$$

where S_{IO} is defined to be the input and output apparent powers of the transformer. However, the apparent power in the transformer winding S_W is

the power in winding ' ac ' $S_{ac} = (V_H - V_L)I_H$

the power in winding ' bc ' $S_{bc} = V_L(I_L - I_H)$

$$S_{bc} = V_L I_L \left(1 - \frac{I_H}{I_L}\right)$$

$$S_{bc} = V_L I_L \left(1 - \frac{N_L}{N_H}\right)$$

$$S_W = S_{IO} \frac{N_H - N_L}{N_H}$$

$$S_{IO} = S_W \frac{N_{SE} + N_C}{N_{SE}}$$

Therefore, the ratio of the apparent power in the primary and the secondary of the autotransformer to the apparent power actually traveling through its windings is

$$\frac{S_{IO}}{S_W} = \frac{N_{SE} + N_C}{N_{SE}} > 1$$

The above Equation describes the apparent power rating advantage of an autotransformer over a conventional two winding transformer.

3.4 Saving of Copper in Autotransformers

Refer to Fig. 3.3b, the weight of copper in *ac* winding (series winding) is proportional to

$$(N_H - N_L)I_H$$

While, the weight of copper in *bc* winding (common winding) is proportional to

$$N_L(I_L - I_H)$$

So, the total weight of copper in autotransformer winding, W_{auto} is proportional to

$$(N_H - N_L)I_H + N_L(I_L - I_H)$$

On the other hand, refer to Fig. 3.3a, for the same transformation, the weight of copper in two winding transformer, W_{2w} is proportional to

$$N_H I_H + N_L I_L$$

Therefore, the ratio of total weight of copper in autotransformer winding to that of two-winding transformer is

$$\frac{W_{auto}}{W_{2w}} = \frac{(N_H - N_L)I_H + N_L(I_L - I_H)}{(N_H I_H + N_L I_L)}$$

$$= \frac{N_H I_H - N_L I_H + N_L I_L - N_L I_H}{(N_H I_H + N_L I_L)}$$

$$= \frac{(N_H I_H + N_L I_L) - 2N_L I_H}{(N_H I_H + N_L I_L)}$$

$$= 1 - \frac{2N_L I_H}{(N_H I_H + N_L I_L)}$$

$$= 1 - \frac{2\, I_H/I_L}{(1 + N_H/N_L \times I_H/I_L)}$$

$$= 1 - \frac{2\, N_L/N_H}{(1 + N_H/N_L \times N_L/N_H)}$$

$$= 1 - N_L/N_H$$

$$= \frac{N_H - N_L}{N_H} < 1$$

Therefore,

$$W_{auto} = \left(1 - \frac{1}{a}\right) W_{2w}$$

Hence, $|\text{saving in copper}| = W_{auto} - W_{2w}$

$$= \frac{1}{a} W_{2w}$$

<u>*Noted that:*</u>

> *The saving in copper weight in an autotransformer is inversely proportional to the turn's ratio of the conventional two winding transformer.*

3.5 Advantages and Disadvantages of Autotransformers

The advantages of an autotransformer connection are:

1. Variable output voltage when a sliding contact is used for the secondary.
2. Increased kVA rating $S_{IO} > S_W$.
3. Saving weight of copper, (an autotransformer requires less copper than a two-winding transformer of similar rating), therefore, an autotransformer has smaller size than a two-winding transformer of the same rating.

4. Lower leakage reactances and lower losses, therefore, an autotransformer operates at a higher efficiency than a two-winding transformer of similar rating.
5. An autotransformer has better voltage regulation than a two-winding transformer of the same rating.

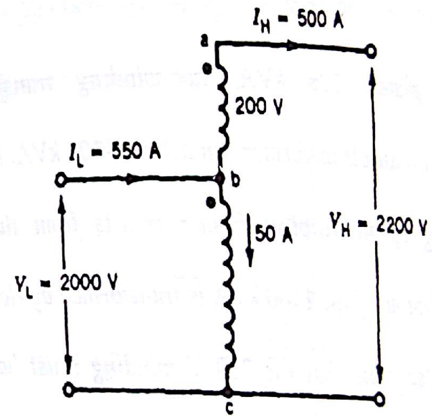
The disadvantages are:

1. A direct connection between the primary and secondary sides and
2. The short-circuit current is much larger than for the two-winding transformer of the same rating. It can be seen from Figure 3.3b that, a short-circuited secondary causes part of the primary also to be short-circuited. This reduces the effective resistance and reactance.

EXAMPLE 3.1

A 1-ϕ, 100 kVA, 2000/200 V two-winding transformer is connected as an autotransformer as shown in Fig. such that more than 2000 V is obtained at the secondary. The portion ab is the 200 V winding, and the portion be is the 2000 V winding. Compute the kVA rating as an autotransformer.

Solution



The current ratings of the windings are

$$I_{ab} = \frac{100{,}000}{200}\text{ A} = 500\text{ A}$$

$$I_{bc} = \frac{100{,}000}{2000} = 50\text{ A}$$

Therefore, for full-load operation of the autotransformer, the terminal currents are:

$$I_H = 500\ \text{A}$$

$$I_L = 500 + 50 = 550\ \text{A}$$

Now, $V_L = 2000$ V and

$$V_H = 2000 + 200 = 2200\ \text{V}$$

Therefore,

$$\text{kVA}|_L = \frac{2000 \times 550}{1000} = 1100$$

$$\text{kVA}|_H = \frac{2200 \times 500}{1000} = 1100$$

A single-phase, 100 kVA, two-winding transformer when connected as an autotransformer can deliver 1100 kVA. Note that this higher rating of an autotransformer results from the conductive connection. Not all of the 1100 kVA is transformed by electromagnetic induction. Also, note that the 200 V winding must have sufficient insulation to withstand a voltage of 2200 V to ground.

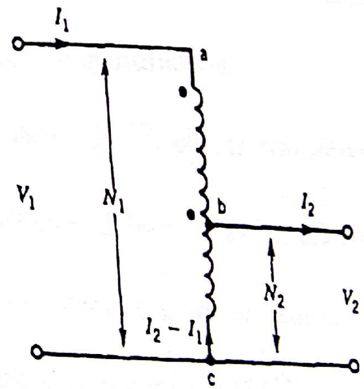
EXAMPLE 3.2

A single phase, 50 kVA, 2400/460 V, 50 Hz transformer has an efficiency of 0.95% when it delivers 45kW at 0.9 power factor. This transformer is connected as an auto-transformer to supply load to a 2400 V circuit from 2860 V source.

(a) Show the transformer connection.

(b) Determine the maximum kVA the autotransformer can supply to 2400 V circuit. (c) Determine the efficiency of the autotransformer for full load at 0.9 power factor.

Solution



(a)

(b) $I_{x,2w} = \dfrac{50*10^3}{2460} = 108.7\ A$

Then, $kVA)_{Auto} = 108.782860 = 310.87\ kW$

(c) $\eta_{2w} = \dfrac{50*10^3*0.9}{50*10^3*0.9+P_i+P_{cu,FL}} = 0.95$

Then, $P_i + P_{cu,FL} = 2368.42\ W$

$\eta_{Auto} = \dfrac{310870*0.9}{310870*0.9+2368.42} = 99.61\ \%$

Review Questions (3)

56. What is an autotransformer? List its advantages and drawbacks.

57. Why can autotransformer handle more power than conventional transformer of the same size?

58. With the aid of necessary equations and figures, explain the principle of copper saving in an autotransformer.

59. Draw sketches showing how a 22-kVA, 2200/1100-V, two-winding transformer can be connected as an autotransformer. Determine (a) the voltage rating, (b) the power rating, (c) the power transferred by conduction, and (d) the power transferred by induction.

60. A 1 ϕ, 10 kVA, 460/ 120 V, 60 Hz transformer has an efficiency of 96% when it delivers 9 kW at 0.9 power factor. This transformer is connected as an autotransformer to supply load to a 460 V circuit from a 580 V source.

    (a) Show the autotransformer connection.

    (b) Determine the maximum kVA the autotransformer can supply to the 460 V circuit.

(c) Determine the efficiency of the autotransformer for full load at 0.9 power factor.

61. Reconnect the windings of a 1ϕ, 3 kVA, 240/120 V, 60-Hz transformer so that it can supply a load at 330 V from a 110 V supply. (a) Show the connection. (b) Determine the maximum kVA the reconnected transformer can deliver.

Chapter 4

Three-Phase Transformers

Chapter 4

Three-Phase Transformers

4.1 Introduction

The transformers used in the electric power systems, including generation, transmission and distribution systems are called three-phase transformers. It requires stepping up or stepping down voltages in the various stages of the power systems. Obviously, both primary and secondary windings, in a three-phase transformer, must each have three phase windings for three phases. The windings can be connected in star as well as in delta connections. It can be noted that, all the phase windings must be identical to make a balanced three-phase system. *A three-phase transformer can be built using two approaches:*

1. By suitably connecting a bank of three single-phase transformers [shown in Fig. 4.1], or
2. By constructing a three-phase transformer on a common magnetic structure [shown in Fig. 4.2].

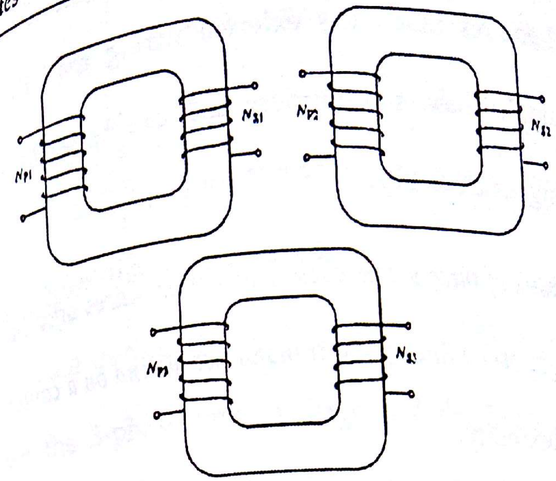


Fig. 4.1. A three-phase transformer bank.

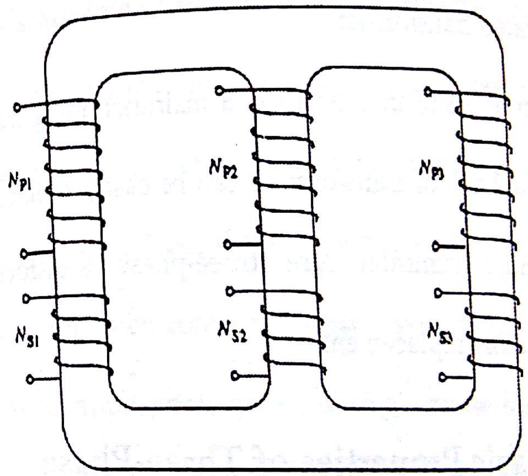


Fig. 4.2. A three-phase transformer in a common core.

Advantages of a three-phase transformer wound on a common core over a bank of three single-phase transformers are:

1. Lighter, 2. Smaller, 3. Cheaper and

4. Requires much less external wiring than the bank of single-phase transformers and can typically achieve a higher efficiency.

The bank of three single-phase transformers does offer the following advantages over a three-phase transformer wound on a common core:

1. Flexibility,
2. In the case of an unbalanced load, one or more transformer in the bank can be replaced by a larger or smaller KVA-rated transformer.
3. In terms of maintenance, a malfunctioning transformer in the bank of transformers can be easily replaced while the entire common core three-phase transformer would require replacement.

4.2 Basic Properties of Three-Phase Transformer Banks

When three single-phase transformers are used to transform a 3-phase voltage, the windings can be connected in several ways. Thus, the primaries may be connected in delta

and the secondaries in wye, or vice versa. As a result, the ratio of the 3-phase input voltage to the 3-phase output voltage depends not only upon the turns ratio of the transformers, but also upon how they are connected.

A 3-phase transformer bank can also produce a phase shift between the 3-phase input voltage and the 3-phase output voltage. The amount of phase shift depends again upon the turns ratio of the transformers, and on how the primaries and secondaries are interconnected. Furthermore, the phase shift feature enables us to change the number of phases. Thus, a 3-phase system can be converted into a 2-phase, a 6-phase, or a 12-phase system. Indeed, if there were a practical application for it, we could even convert a 3-phase system into a 5-phase system by an appropriate choice of single-phase transformers and interconnections.

In making the various connections, it is important to observe transformer polarities. An error in polarity may produce a short-circuit or unbalance the line voltages and currents.

4.3 Transformer Winding Connection Designations

1. **First Symbol: for High Voltage:** Always capital letters such as: D=Delta, S=Star, N=Neutral, Z=Zigzag
2. **Second Symbol: for Low voltage:** Always Small letters such as d=Delta, s=Star, n=Neutral, z=Zigzag.
3. **Third Symbol:** Phase displacement expressed as the clock hour number. Because there are 12 hours on a clock, and a circle consists out of 360°, each hour represents 30°. Thus 1 = 30°, 2 = 60°, 3 = 90°, 6 = 180° and 12 = 0° or 360°.

Example - Dyn11

Transformer has a delta connected primary winding (D) a star connected secondary (y) with the star point brought out (n) and a phase-shift of 30° leading (11).

4.4 Basic Idea of Winding

As illustrated in the single-phase transformer, an ac voltage applied to a coil will induce a voltage in a second coil where the

two are linked by a magnetic path. The phase relationship of the two voltages depends upon which ways round the coils are connected. The voltages either will be in-phase or in anti-phase (displaced by 180 degrees).

When 3 coils are used in a 3-phase transformer winding, a number of options exist. The coil voltages can be in-phase or anti-phase as above with the coils connected in star or delta and, in the case of a star-winding, have the star point (neutral) brought out to an external terminal or not.

In the three-phase system, there are six ways to wire the star as well as the delta winding. These ways determine the phase relation between the primary and secondary windings. In the following subsections these ways to wire the star and delta connections are presented.

4.4.1 Six Ways to wire Star Winding:

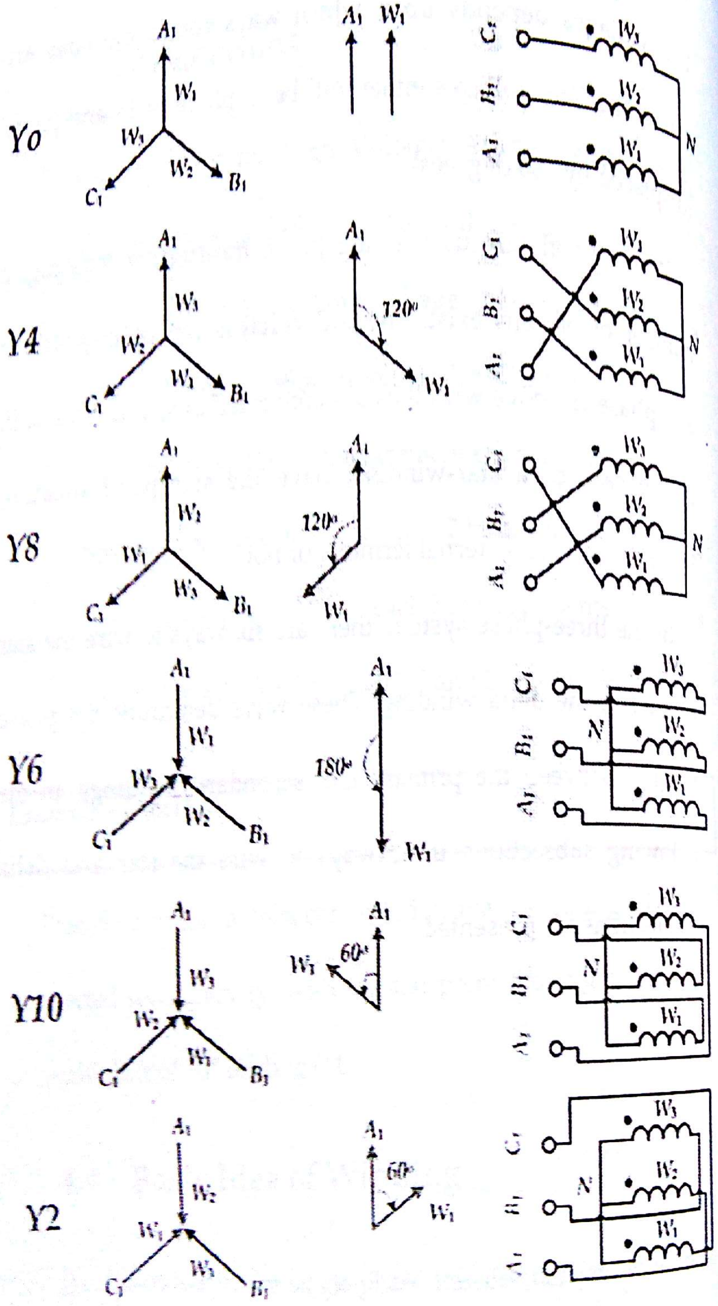


Fig. 4.3. Six Ways to wire Star Winding

4.4.2 Six ...

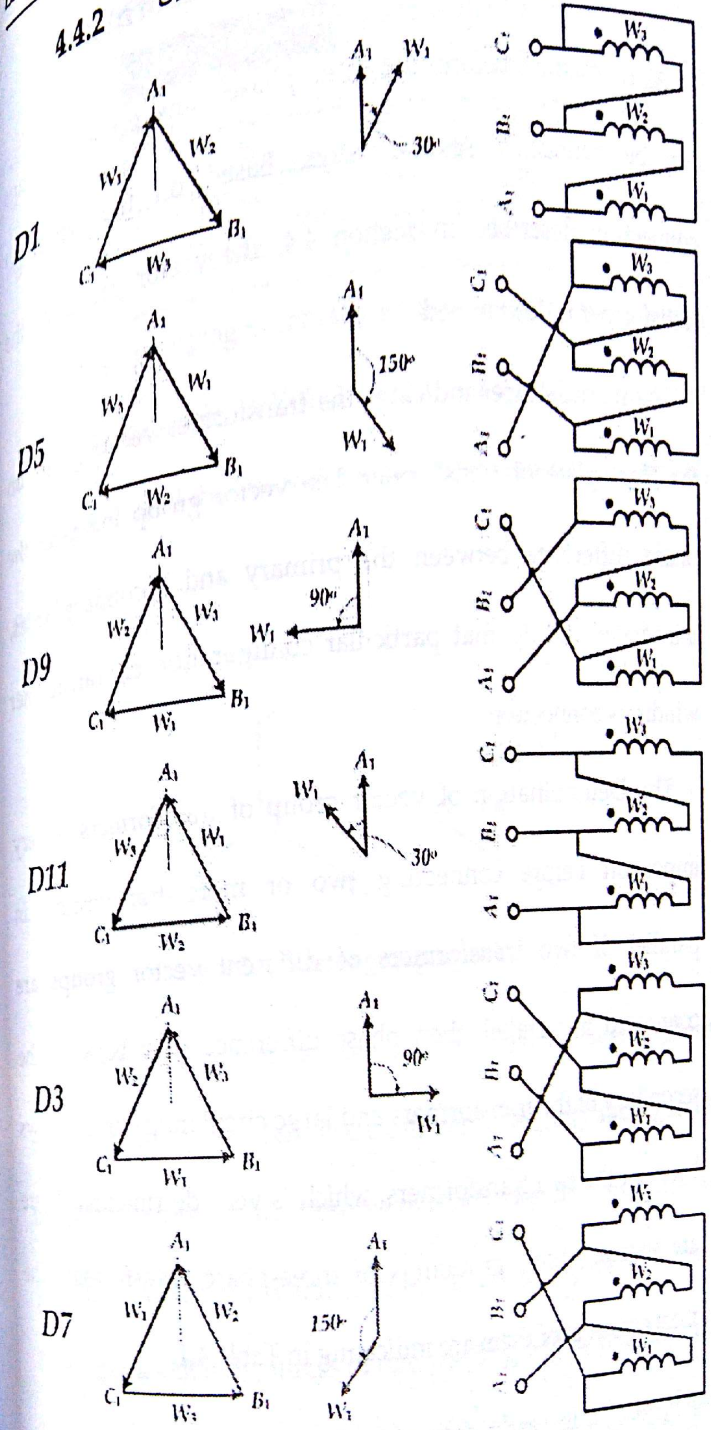


Fig. 4.4. Six Ways to wire Delta Winding

4.5 Vector Grouping of Transformer

As mentioned before, the three phase transformer windings can be connected several ways. Based on the windings' connection described in section 4.4, the vector group of the transformer is determined.

The manufacturer indicates the transformer vector group on the Nameplate of transformer. The vector group indicates the phase difference between the primary and secondary sides, introduced due to that particular configuration of transformer windings connection.

The Determination of vector group of transformers is very important before connecting two or more transformers in parallel. If two transformers of different vector groups are connected in parallel, then phase difference exist between the secondary of the transformers and large circulating current flows between the two transformers, which is very detrimental. There are several vector groupings of three-phase transformer. The most common of them are indicating in Table 4.1.

Table 4.1 Common vector grouping of the three-phase transformer

Phase Shift (Deg.)	Connection	
0	Yy0	Dd0
30 lag	Yd1	Dy1
180 lag	Yy6	Dd6
30 lead	Yd11	Dy11

The phase grouping is actually related to the circular rotation of clock. For example, in Yd1, 1 means that the high voltage star winding will lead the low voltage winding delta winding by 30°. See Fig. 4.5,

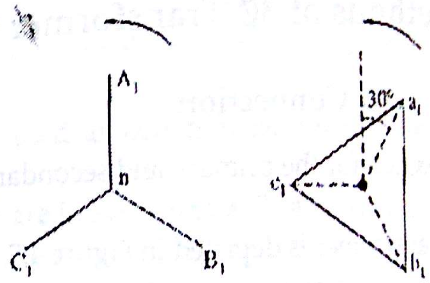


Fig. 4.5. Vector grouping of Yd1

Figure 4.6 shows a circle like watch where 0 indicates 0° phase difference, 1 indicates 30° phase difference lagging, and so on as indicated before. If we observe Fig. 4.5, we find that A_1n is parallel to a_1b_1, B_1n is parallel to b_1c_1 and C_1n is parallel to c_1a_1. This signifies that the fluxes of one winding will be fully mutual linked with the other winding in each phase of a three-phase

system. Thus, full utilization of the input flux will be possible in this type of arrangement. That is why; Yd1 is one of the important vector groupings.

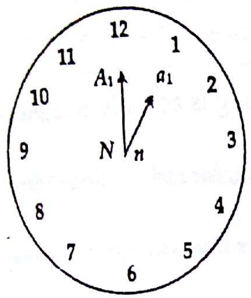


Fig. 4.6. Phase difference of vector grouping of Yd1

4.6 Methods of 3Ø Transformer Connection

4.6.1 Y-y Connection

A Y-y connection for the primary and secondary windings of a three-phase transformer is depicted in Figure 4.7.

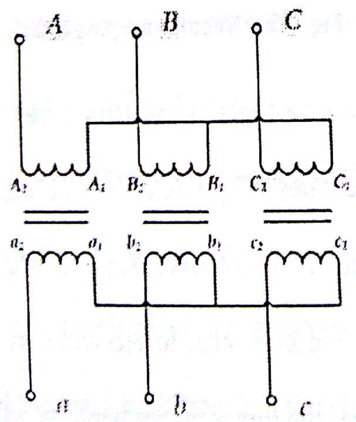


Fig. 4.7. Y-y connected three-phase transformer (zero-degree connection)

As shown in Fig. 4.6, ...

transformer is formed by connecting one terminal of each phase of individual side, together. The primary windings are denoted by A, B, and C letters, while the secondary windings are denoted by a, b, and c letters. There are two different ways to connect the Y-y transformer according to how the primaries winding or the secondaries are connected. The resulted two connections are called Yy0 and Yy6.

In Yy0 (zero phase displacement between primary and secondary), the terminal with suffix 1 in both primary and secondary are used as common terminal, voltages of primary and secondary are in same phase. That is why this connection is called zero-degree connection (0°- connection) or Yy0.

The phasor diagram of the phase voltages and the vector group of this connection are shown in Fig. 4.8.

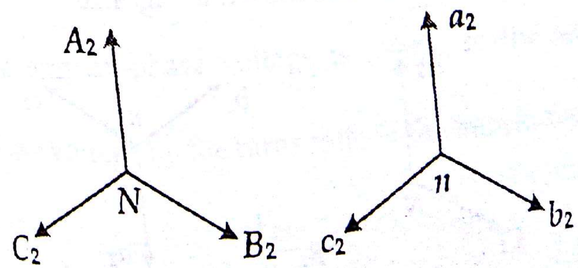


Fig. 4.8. Phasor diagram and Vector group of Yy0 transformer

In $Yy6$ (180-degree phase displaced), the terminals with suffix 1 are connected together in primary as common point and the terminals with suffix 2 in secondary are connected together as common point [see Fig. 4.9], the voltages in primary and secondary will be in opposite phase. Hence, the transformer is called 180°-connection, of three phase transformer or $Yy6$.

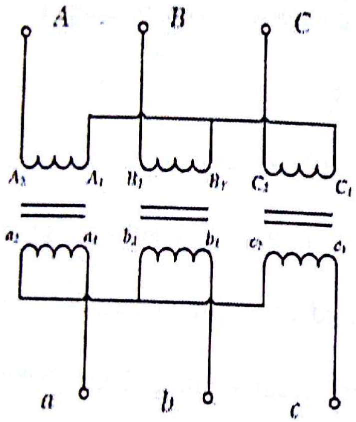


Fig. 4.9. $Yy6$ connected three-phase transformer

The phasor diagram of the phase voltages and the vector group of this connection are shown in Fig. 4.10.

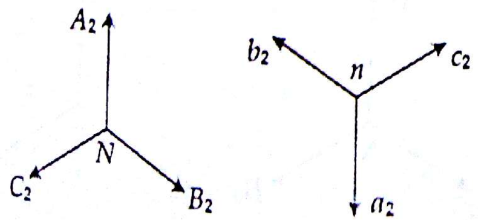


Fig. 4.10. Phasor diagram and Vector group of $Yy6$ transformer

Analysis of Y-y transformer

Refer to the Y-Y connection of the three-phase transformer shown in Fig. 4.11.

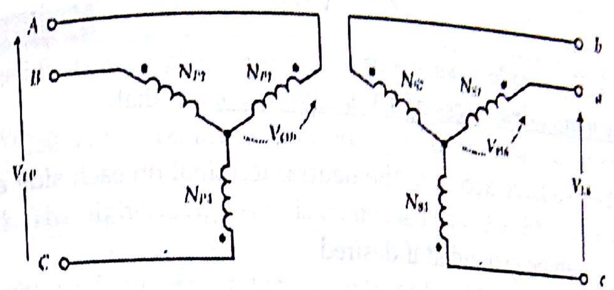


Fig. 4.11. Wiring diagram of Y-Y connected 3Ø transformer

- The primary voltage on each line-to-line of the three-phase transformer is given by

$$V_{LP} = \sqrt{3}V_{\phi P}$$

- Also, the secondary voltage on each line-to-line of the three-phase transformer is given by

$$V_{LS} = \sqrt{3}V_{\phi S}$$

- The primary-phase voltage is related to the secondary-phase voltage by the turns ratio of the transformer

$$\frac{V_{\phi P}}{V_{\phi S}} = a$$

- Therefore, overall the voltage ratio on the Y-Y connected three-phase transformer is

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\phi P}}{\sqrt{3}V_{\phi S}} = a$$

<u>*The main advantages of a Y-y connection*</u> are that:

1- We have access to the neutral terminal on each side and it can be grounded if desired.

2- The electrical insulation is stressed only to about 58% of the line voltage in a Y-connected transformer.

<u>*The main disadvantages of a Y-y connection*</u> are that:

1- Without grounding the neutral terminals, the *Y-y* operation is satisfactory only when the three-phase load is balanced. Therefore, it is rarely used due to the problems with unbalanced loads.

2- Since most of the transformers are designed to operate at or above the knee of the magnetization curve, such a design causes the induced emfs and currents to be distorted. The reason is as follows: Although the excitation currents are still 120° out of phase with respect to each other, their waveforms are no more sinusoidal. These

currents, therefore, do not add up to zero. If the neutral is not grounded, these currents are forced to add up to zero. Thus, they affect the waveforms of the induced emfs.

Tutorial 4.1

Three identical single-phase transformers, each of rating 20 kVA, 2300/230 V, 60 Hz, are connected *Yy* to form a 3φ transformer bank. The high-voltage side is connected to a 3φ, 4000 V, 60 Hz supply, and the secondary is left open. The neutral of the primary is not connected to the neutral of the supply. The voltage between the primary neutral and the supply neutral is measured to be 1200 V.

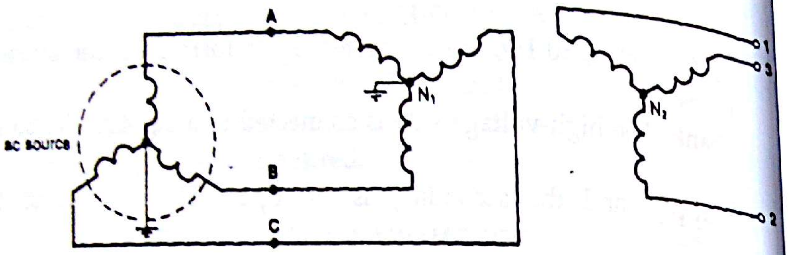
a) Describe the voltage waveform between primary neutral and supply neutral. Neglect harmonics higher than third.

b) Determine the ratio of (i) phase voltages of the two sides and (ii) line voltages of the two sides.

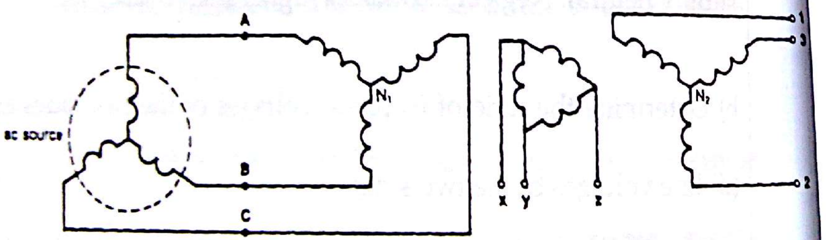
c) Determine the ratio of the rms line-to-line voltage to the rms line-to-neutral voltage on each side.

Tertiary Windings

In order to prevent the distortion in the Y-y connection, the neutral of the primary can be connected to the neutral of the source, usually by way of the ground as follows;



Another way is to provide each transformer with a third winding, called tertiary winding. The tertiary windings of the three transformers are connected in delta as shown in the following figure.



They often provide the substation service voltage where the transformers are installed.

4.6.2 Y-d Connection

A Y-d connection for the primary and secondary windings of a three-phase transformer is depicted in Figure 4.12.

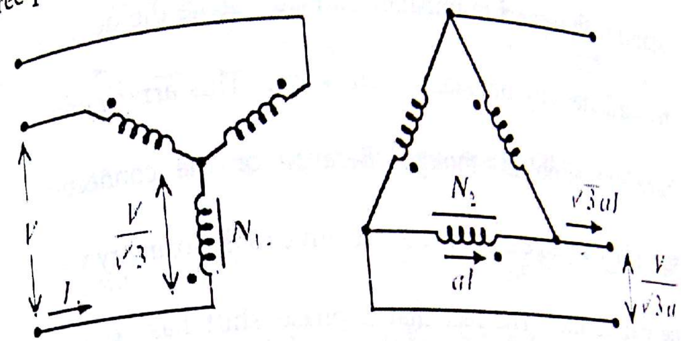


Fig. 4.12. Wiring diagram of Y-d connected 3Ø transformer

In this connection, the primary line voltage is related to the primary phase voltage by $V_{LP} = \sqrt{3}V_{\phi P}$, while the secondary line voltage is equal to the secondary phase voltage $V_{LS} = V_{\phi S}$. The voltage ratio of each phase is

$$\frac{V_{\phi P}}{V_{\phi S}} = a$$

- Therefore, overall the voltage ratio on the Y-d connected three-phase transformer is

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\phi P}}{V_{\phi S}} = \sqrt{3}a$$

The Y-d connection has no problem with third-harmonic components in its voltages, since they are consumed in a circulating current on the delta side. This connection is also more stable with respect to unbalanced loads, since the delta partially redistributes any imbalance that occurs. This arrangement does have one problem, though. Because of the connection, the secondary voltage is shifted 30° relative to the primary voltage of the transformer. The fact that a phase shift has occurred can cause problems in paralleling the secondaries of two transformer banks together. The phase angles of transformer secondaries must be equal if they are to be paralleled, which means that attention must be paid to the direction of the 30° phase shift occurring in each transformer bank to be paralleled together.

4.6.3 *D-y* Connection

A D-y connection for the primary and secondary windings of a three-phase transformer is depicted in Figure 4.13.

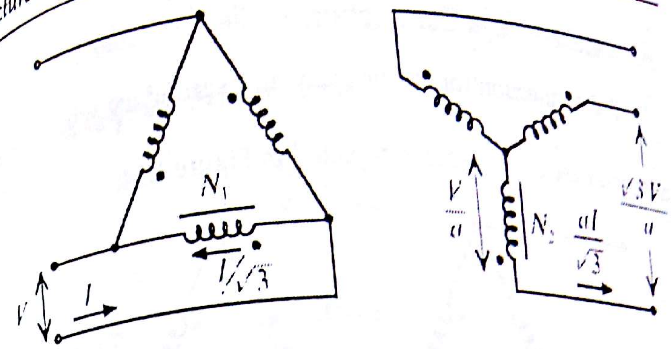


Fig. 4.13. Wiring diagram of D-y connected 3Ø transformer

In this connection, the primary line voltage is related to the primary phase voltage by $V_{LP} = V_{\emptyset P}$, while the secondary line voltage is equal to the secondary phase voltage $V_{LS} = \sqrt{3}V_{\emptyset S}$. The voltage ratio of each phase is

$$\frac{V_{\emptyset P}}{V_{\emptyset S}} = a$$

- Therefore, overall the voltage ratio on the D-y connected three-phase transformer is

$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\emptyset P}}{\sqrt{3}V_{\emptyset S}} = \frac{a}{\sqrt{3}}$$

This connection has the same advantages and the same phase shift as the Y-d transformer. The connection shown in Fig. 4.13 makes the secondary voltage lag the primary voltage by 30°, as

4.6.4 D-d Connection

A D-d connection for the primary and secondary windings of a three-phase transformer is depicted in Figure 4.14.

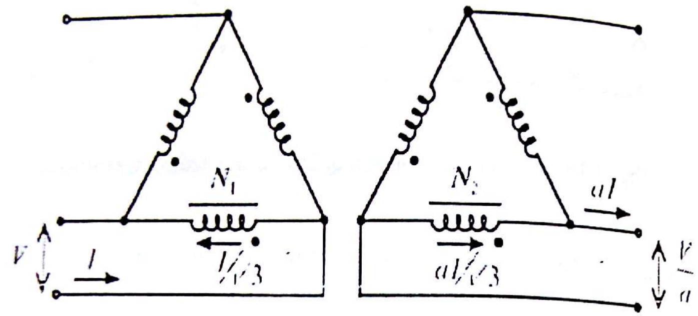


Fig. 4.14. Wiring diagram of D-d connected 3Ø transformer

- The primary voltage on each phase of the three-phase transformer is given by

$$V_{LP} = V_{\phi P}$$

- Also, the secondary voltage on each phase of the three-phase transformer is given by

$$V_{LS} = V_{\phi S}$$

- The primary-phase voltage is related to the secondary-phase voltage by the turns ratio of the transformer

$$\frac{V_{\phi P}}{V_{\phi S}} = a$$

- Therefore, overall the voltage ratio on the D-d connected three-phase transformer is

$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{V_{\phi S}} = a$$

4.6.5 V Connection

In the Δ-Δ connection of three single-phase transformers, one transformer can be removed and the system can still deliver three-phase power to a three-phase load. This configuration is known as an open-delta or V connection. It may be employed in an emergency situation when one transformer must be removed for repair and continuity of service is required.

Consider Fig. 4.15, in which one transformer, shown dotted, is removed. For simplicity, the load is considered to be Y-connected.

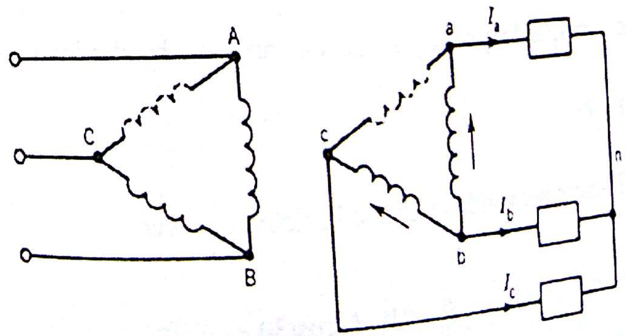


Fig. 4.15. Open-delta connection

Figure 4.16 shows the phasor diagram for voltages and currents. Here V_{AB}, V_{BC}, and V_{CA} represent the line-to-line voltages of the primary; V_{ab}, V_{bc}, and V_{ca} represent the line-to-line voltages of the secondary; and V_{an}, V_{bn}, and V_{cn} represent the phase voltages of the load.

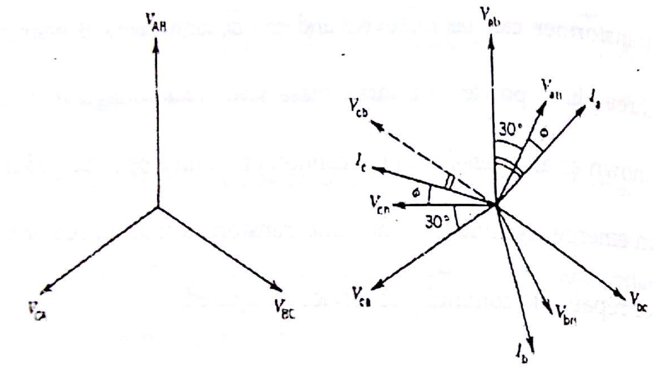


Fig. 4.16. Phasor diagram of open-delta connection

For an inductive load, the load currents I_a, I_b, and I_c will lag the corresponding voltages V_{an}, V_{bn}, and V_{cn} by the load phase angle φ.

Transformer windings ab and bc deliver power

$$P_{ab} = V_{ab} I_a \cos(30 + \varphi)$$

$$P_{bc} = V_{cb} I_c \cos(30 - \varphi)$$

where $|V_{ab}| = |V_{cb}| = V$, voltage rating of the transformer secondary winding, $|I_a| = |I_c| = I$, current rating of the transformer secondary winding and $\varphi = 0$ for a resistive load.

Power delivered to the load by the V connection is

$$P_V = P_{ab} + P_{bc} = 2VI\cos 30^\circ$$

With all three transformers connected in delta, the power delivered is

$$P_\Delta = 3VI$$

Thus

$$\frac{P_V}{P_\Delta} = \frac{2\cos 30^\circ}{3} = 0.58$$

The V connection is capable of delivering 58% power without overloading the transformer.

Comparison between three-phase transformer connections

Y-Δ:	Commonly used in a **step-down** transformer, Y-connection on the HV side reduces insulation costs; the neutral point on the HV side can be grounded, stable with respect to unbalanced loads.
Δ-Y:	Commonly used in a **step-up** transformer for the same reasons as above.
Δ- Δ:	Offers the advantage that one of the transformers can be removed while the remaining two transformers can deliver three-phase power at 58% of the original bank
Y-Y:	The main advantage of a Y/Y connection is that we have access to the neutral terminal on each side and it can be grounded if desired. Without grounding the neutral terminals, the Y/Y operation is satisfactory only when the three-phase load is balanced. The electrical insulation is stressed only to about 58% of the line voltage in a Y-connected transformer. Rarely used, problems with unbalanced loads.

Example

Three single-phase transformers are connected in delta-delta to step down a line voltage of 138 kV to 4160 V to supply power to a manufacturing plant. The plant draws 21 MW at a lagging power factor of 86 percent. Calculate:

a) The apparent power drawn by the plant

b) The apparent power furnished by the HV line

c) The current in the HV lines

d) The current in the LV lines

e) The currents in the primary and secondary windings of each transformer.

Solution:

a. The appearent power drawn by the plant is:

$S = P/\cos\varphi = 21/0.86 = 24.4$ MVA

b. The transformer bank itself absorbs a negligible amount of active and reactive power because the I^2R losses and the reactive power associated with the mutual flux and the leakage fluxes are small. It follows that the apparent power furnished by the HV line is also 24.4 MVA.

c.The current in each HV line is:-

$$I_1 = \frac{S}{\sqrt{3} * V_1} = \frac{24.4*10^6}{\sqrt{3}*13800} = 102\ A$$

d.The current in the LV lines is:-

$$I_2 = \frac{S}{\sqrt{3}V_2} = \frac{24.4*10^6}{\sqrt{3}*4160} = 3386\ A$$

e. The current in each primary winding is:

$$I_p = \frac{102}{\sqrt{3}} = 58.9\ A$$

The current in each secondary winding is:

$$I_S = \frac{3386}{\sqrt{3}} = 1955\ A$$

f. Because the plant load is balanced, each transformer carries one-third of the total load, or 24.4/3 = 8.13 MVA.

The individual transformer load can also be obtained by multiplying the primary voltage times the primary current:

$$S = E_p I_p = 138000 * 58.9 = 8.13\,MVA$$

Note that we can calculate the line currents and the currents in the transformer windings even though we do not know how the 3-phase load is connected. In effect, the plant load is composed of

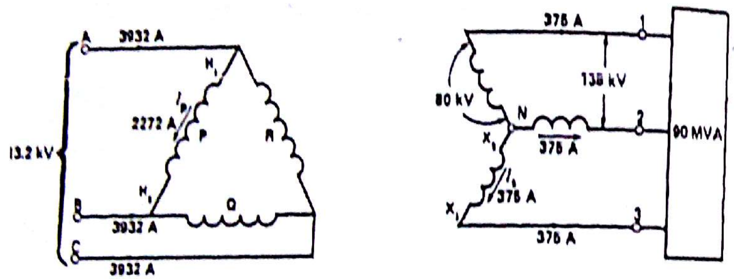
hundreds of individual loads, some of which are connected in delta, others in wye. Furthermore, some are single-phase loads operating at much lower voltages than 4160 V, powered by smaller transformers located inside the plant. The sum total of these loads usually results in a reasonably well-balanced 3-phase load, represented by the box.

***Example** Three single-phase step-up transformers rated at 90 MVA, 13.2 kV/80 kV are connected in delta-wye on a 13.2 kV transmission line. If they feed a 90 MVA load, calculate the following:*

a. The secondary line voltage

b. The currents in the transformer windings

c. The incoming and outgoing transmission line currents



Solution

The easiest way to solve this problem is to consider the windings of only one transformer, say, transformer P.

a. The voltage across the primary winding is obviously 13.2 kV

The voltage across the secondary is, therefore, 80 kV.

The voltage between the outgoing lines 1, 2, and 3 is:

$$V_2 = 80 * \sqrt{3} = 139\ kV$$

$$E_s = 80\sqrt{3} = 139\ \text{kV}$$

b. The load carried by each transformer is

$$S = 90/3 = 30\ \text{MVA}$$

The current in the primary winding is

$$I_p = 30\ \text{MVA}/13.2\ \text{kV} = 2273\ \text{A}$$

The current in the secondary winding is

$$I_s = 30\ \text{MVA}/80\ \text{kV} = 375\ \text{A}$$

c. The current in each incoming line A, B, C is

$$I = 2273\sqrt{3} = 3937\ \text{A}$$

The current in each outgoing line 1, 2, 3 is

$$I = 375\ \text{A}$$

Example

Three single phase, 30 kVA, 2400/240 V, 50 Hz transformers are connected to form 3 φ, 2400/416 V transformer bank. The equivalent impedance of each transformer referred to the high voltage side is 1.5+j2 Ω. The transformer delivers 60 kW at 0.75 power factor (leading).

(a) Draw schematic diagram showing the transformer connection.

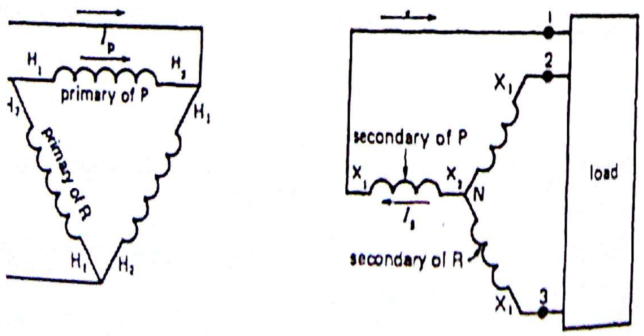
(b) Determine the transformer winding current

(c) Determine the primary voltage.

(d) Determine the voltage regulation.

Solution:

(a)



(b) $kVA = \frac{60}{\ldots} = 80kVA$

$$I_s = \frac{80*10^3}{\sqrt{3}*416} = 111.029\ A$$

$$a = \frac{2400}{240} = 10$$

$$I_{1ph} = \frac{111.029}{10} = 11.103A$$

$$I_{1L} = 11.103*\sqrt{3} = 19.231\ A$$

$$V_2' = 2400\angle 0^o\ V\ ,\ I_2' = 11.103\angle 41.41^o\ A$$

$$V_1 = V_2' + I_2' * (Z_{eq1})$$

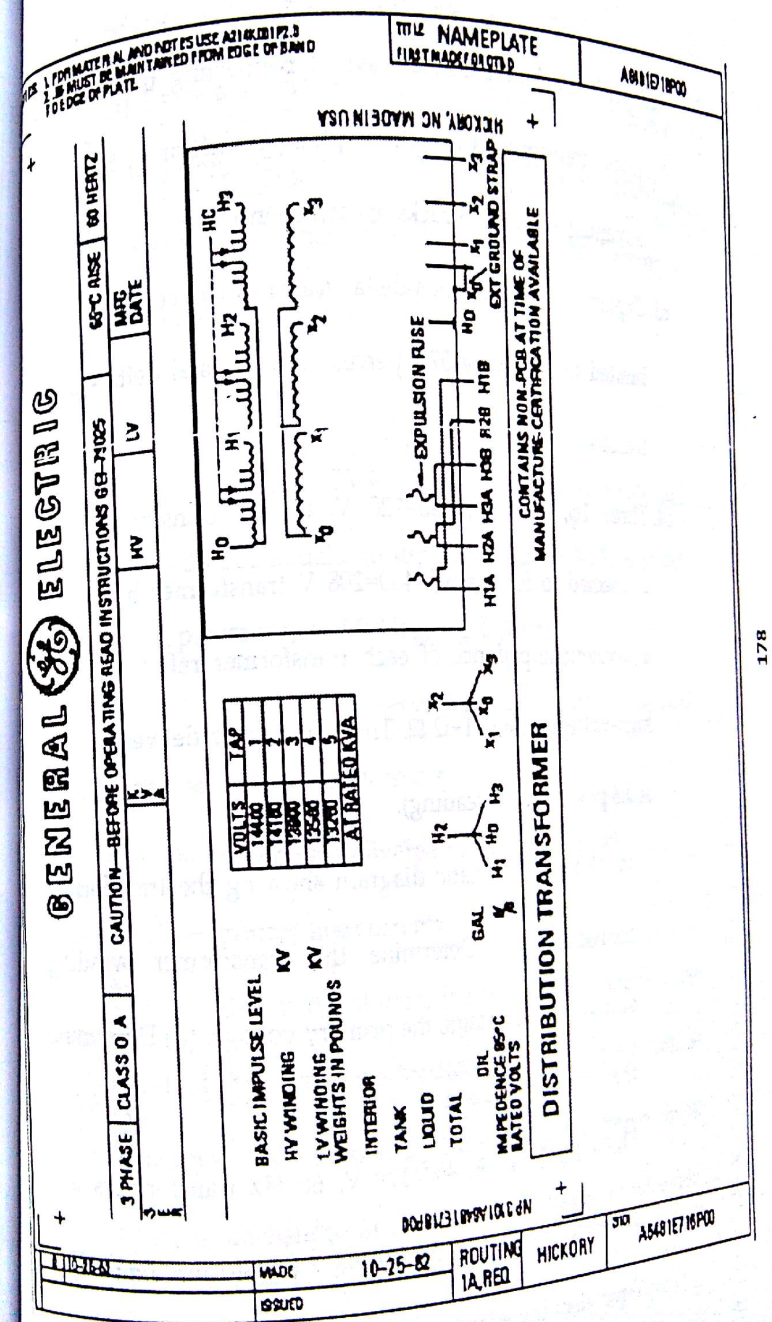
$$= 2400\angle 0 + 11.103\angle 41.41^o * (1.5 + j2) = 2397.96\angle 0.66^o V$$

$$V_R = \frac{V_1 - V_2'}{V_2'} * 100$$

$$= \frac{2397.96 - 2400}{2400} * 100 = -0.0875\%$$

4.7 The Transformer Nameplate

A typical nameplate from a distribution transformer is shown in the following figure. The information on such a nameplate includes rated voltage, rated kVA, rated frequency, and the transformer per-unit series impedance. It also shows the voltage ratings for each tap on the transformer and the wiring schematic of the transformer. Nameplates such as the one shown also typically include the transformer type designation and references to its operating instructions.



Sample of Review Questions (4)

62. Sketch the four possible ways of connecting three single-phase transformers as a three-phase transformer. State the advantages and drawbacks of each connection.

63. Explain why the open-delta transformer connection is limited to supplying 57.7 percent of a normal delta - delta transformer bank's load.

64. Three 1φ, 10 kVA, 460=120 V, 60 Hz transformers are connected to form a 3φ, 460=208 V transformer bank. The equivalent impedance of each transformer referred to the high-voltage side is 1+j2 Ω. The transformer delivers 20 kW at 0.8 power factor (leading).

(a) Draw a schematic diagram showing the transformer connection. (b) Determine the transformer winding current. (c) Determine the primary voltage. (d) Determine the voltage regulation.

Three 1¢, 10 kVA, 460/120 V, 60 Hz transformers are connected to form a 3ϕ 460/208 V transformer bank. The equivalent impedance of each transformer referred to the

high-voltage side is 1.0 + j2.0 Ω. The transformer delivers 20 kW at 0.8 power factor (leading).

(a) Draw a schematic diagram showing the transformer connection. (b) Determine the transformer winding current. (c) Determine the primary voltage. (d) Determine the voltage regulation.

65. Two identical 250 kVA, 230=460 V transformers are connected in open delta to supply a balanced 3φ load at 460 V and a power factor of 0.8 lagging. Determine

(a) The maximum secondary line current without overloading the transformers.

(b) The real power delivered by each transformer.

(c) The primary line currents.

(d) If a similar transformer is now added to complete the Δ, find the percentage increase in real power that can be supplied. Assume that the load voltage and power factor remain unchanged at 460 V and 0.8 lagging, respectively.

Chapter 5

Instrument Transformers

Chapter 5

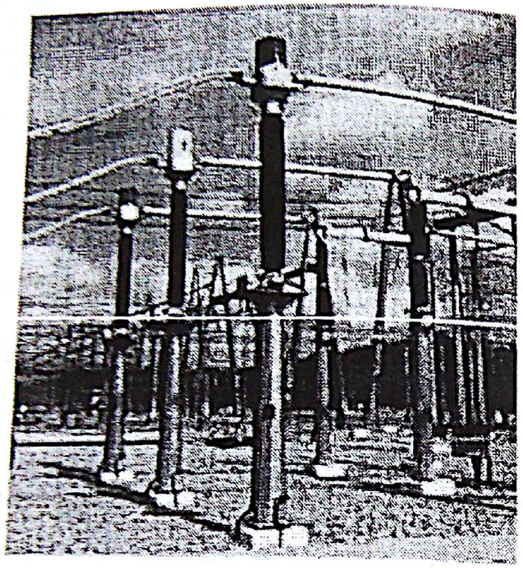
Instrument Transformers

5.1 Introduction of Instrument Transformers

Instrument Transformers are used in AC system for measurement of electrical quantities i.e. voltage, current, power, energy, power factor, frequency. Instrument transformers are also used with protective relays for protection of power system. Basic function of Instrument transformers is to step down the AC System voltage and current. The voltage and current level of power system is very high. It is very difficult and costly to design the measuring instruments for measurement of such high level voltage and current. Generally measuring instruments are designed for 5 A and 110 V.

The measurement of such very large electrical quantities, can be made possible by using the Instrument transformers with these small rating measuring instruments. Therefore, these

instrument transformers are very popular in modern power system.



5.2 Advantages of Instrument Transformers

1. The large voltage and current of AC Power system can be measured by using small rating measuring instrument i.e. 5 A, 110 - 120 V.

2. By using the instrument transformers, measuring instruments can be standardized. Which results in reduction of cost of measuring instruments. More ever the

damaged measuring instruments can be replaced easy with healthy standardized measuring instruments.

3. Instrument transformers provide electrical isolation between high voltage power circuit and measuring instruments. Which reduces the electrical insulation requirement for measuring instruments and protective circuits and assures the safety of operators.

4. Several measuring instruments can be connected through a single transformer to power system.

5. Due to low voltage and current level in measuring and protective circuit, there is low power consumption in measuring and protective circuits.

5.3 Types of Instrument Transformers

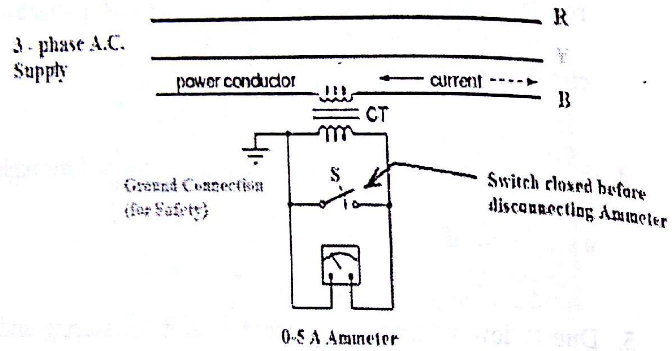
Instrument transformers are of two types -

1. Current Transformer (C.T.)

2. Potential Transformer (P.T.)

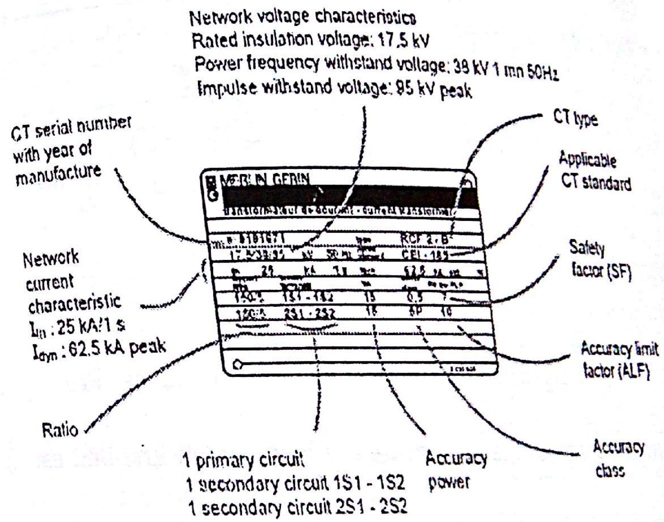
5.3.1 Current Transformer (C.T.)

Current transformer is used to step down the current of power system to a lower level to make it feasible to be measured by small rating Ammeter (i.e. 5A ammeter). A typical connection diagram of a current transformer is shown in figure below.



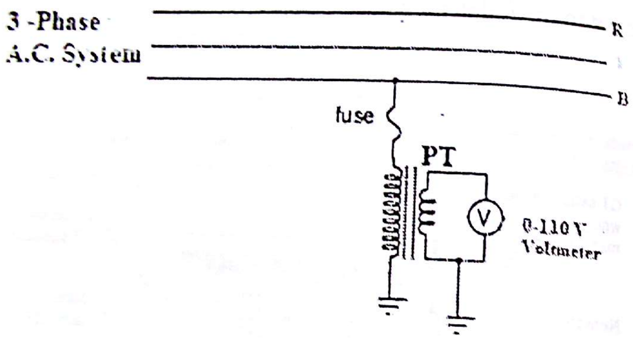
Primary of C.T. is having very few turns. Sometimes bar primary is also used. Primary is connected in series with the power circuit. Therefore, sometimes it also called series transformer. The secondary is having large no. of turns. Secondary is connected directly to an ammeter. As the ammeter is having very small resistance. Hence, the secondary of current transformer operates almost in short circuited condition. One terminal of secondary is earthed to avoid the large voltage on

secondary with respect to earth. Which in turns reduce the chances of insulation breakdown and protect the operator against high voltage. More ever before disconnecting the ammeter, secondary is short circuited through a switch 'S' as shown in figure above to avoid the high voltage build up across the secondary. A typical nameplate of the current transformer is shown in the following figure.



5.3.2 Potential Transformer (P.T.)

Potential transformer is used to step down the voltage of power system to a lower level to make is feasible to be measured by small rating voltmeter i.e. 110 - 120 V voltmeter. A typical connection diagram of a potential transformer is showing figure below.



Primary of P.T. is having large no. of turns. Primary is connected across the line (generally between on line and earth). Hence, sometimes it is also called the parallel transformer. Secondary of P.T. is having few turns and connected directly to a voltmeter. As the voltmeter is having large resistance. Hence, the secondary of a P.T. operates almost in open circuited condition. One terminal of secondary of P.T. is earthed to maintain the

secondary voltage with respect to earth, which assures the safety of operators.

5.3.3 Difference between C.T. and P.T.

Few differences between C.T. and P.T. are listed below -

	Current Transformer (C.T.)	Potential Transformer (P.T.)
1	Connected in series with power circuit.	Connected in Parallel with Power circuit.
2	Secondary is connected to Ammeter.	Secondary is connected to Voltmeter.
3	Secondary works almost in short circuited condition.	Secondary works almost in open circuited condition.
4	Primary current depends on power circuit current.	Primary current depends on secondary burden.
5	Primary current and excitation vary over wide range with change of power circuit current	Primary current and excitation variation are restricted to a small range.
6	One terminal of secondary is earthed to avoid the insulation break down.	One terminal of secondary can be earthed for Safety.
7	Secondary is never be open circuited.	Secondary can be used in open circuit condition.

Sample of Review Questions (5)

66. What is a potential transformer? How is it used?

67. What is a current transformer? How is it used?

Tanta University
Faculty of Engineering
Electrical Power and Machines
Engineering Dept.

Final Exam – First Semester 2015-2016

Course: EPM3111(Electrical Machines 2)	Time allowed: 3 hours
Year: 3rd Electrical Power and Machines Eng.	Date: 14/1/2016
No. of Pages: 3	Total Score: 120

Attempt to solve the following questions.
Answers should be supported by sketches as you can

Question 1 — 10 Points

Tick the correct answer for the following statements:

(1) Under operating conditions, the secondary of a current transformer is always short-circuited to
- (a) prevent a core saturation
- (b) avoid high-voltages on the primary
- (c) avoid high-voltages on the secondary
- (d) none of the above

(2) For parallel operation of transformers, the per-unit impedances of the transformer must be (based on their own kVA rating)
- (a) proportional to ratings
- (b) equal
- (c) inversely proportional to ratings
- (d) none of these

(3) If the secondary winding of the ideal transformer has 40 turns, the number of turns in the primary winding for maximum power transfer to the load will be .... when the supply and load has impedances of 2Ω and 8Ω respectively.
- (a) 20
- (b) 40
- (c) ≈14
- (d) 80

(4) A 2 kVA transformer has iron loss of 150 W and full-load copper loss of 250 W. The maximum efficiency of the transformer would occur when the total loss and per-unit loading are
- (a) 400W, 0.775
- (b) 400W, 0.6
- (c) 300W, 0.775
- (d) none of these

(5) Power transformers are usually designed to have maximum efficiency at
- (a) 50% of full-load
- (b) 85% of full-load
- (c) near full-load
- (d) near no-load

(6) The voltage regulation of transformer at 50% of full-load and 0.85 power factor lagging is 2.5%. The voltage regulation at full-load and 0.8 power factor lagging may be
- (a) -2.5%
- (b) 4.5%
- (c) 2.5%
- (d) 0%

(7) Can a 50 Hz transformer be used for 25 Hz, if the input voltage is maintained constant at the rated value corresponding to 50 Hz?
- (a) Yes. As the voltage is constant, current levels will not change.
- (b) No. Owing to decreased reactance of transformer, input current will be doubled at load.
- (c) No. Flux will be doubled, which will drive the core to excessive saturation.
- (d) Yes. At constant voltage, insulation will not be overstressed.

(8) A 480/120 V, 5 kVA, two-winding transformer is to be used as an autotransformer to supply power at 480 V from 600 V supply. The kVA rating of the autotransformer will be
- (a) 5
- (b) 25
- (c) 1.25
- (d) 15

(9) The open-delta connection of the three-phase transformer is capable of delivering a power reduced by ________ without overloading the transformer.
- (a) 58%
- (b) 80%
- (c) 42%
- (d) 50%

(10) If the transformer is stepping-up then for performing short circuit test meter is connected to the
- (a) secondary
- (b) primary
- (c) Any side
- (d) test cannot be performed on step up transformer

Question 2 — 40 Points

A 25 kVA, 220/440 V, 60 Hz transformer gave the following test results: open-circuit test (220V, 9.5A, 650W) and short-circuit test (37.5V, 55A, 950W).

(a) Derive the approximate equivalent circuit of this transformer referred to low-voltage side.	4
(b) If the transformer is connected to a load whose power-factor varies. Determine worst-case voltage-regulation at *full-load conditions* and draw the corresponding phasor diagram.	4
(c) Find the transformer voltage-regulation at 50% of full-load condition and 0.8 power factor leading and draw the corresponding phasor-diagram.	4
(d) Determine the total-losses and efficiency when the transformer delivers full-load at rated-voltage and 0.8 power-factor leading. (*Don't use the parameters of the equivalent circuit*)	4
(e) Determine the per-unit loading of the transformer at which the efficiency is a maximum and calculate this efficiency at unity power factor.	4

(f) *If the transformer has the following load-cycle:*

No load for 6 hours

70% full load for 10 hours at 0.8 lagging power factor

90% full load for 8 hours at 0.9 leading power factor

Determine the ALL-DAY efficiency of the transformer. — 4

(g) Taking the transformer rating as a base, derive the approximate equivalent circuit of the transformer in per-unit. — 4

(h) Recalculate part (d) using the per-unit values. — 4

(i) Draw a sketch showing how this transformer can be reconnected as a 660/440V autotransformer. Also, determine (I) the power rating S_{IO}, (II) the power transferred by conduction, (III) the power transferred by induction, and (IV) the saving in copper weight — 8

Question 3 — **30 Points**

(a) What are the conditions for satisfying parallel operation of three-phase transformers? — 5

(b) Explain why the open-delta transformer connection is limited to supplying 57.7 % of a normal D-d transformer bank's load.

(c) A three-phase transformer bank is to handle 500-kVA and have a 34.5/11-kV voltage ratio. (I) **Draw the following table in your answer and Fill it with the rating of each individual single-phase transformer in the bank** for the given connections.

Connection	Primary voltage	Secondary voltage	Apparent power	Turns ratio
Y-y				
Y-d				
D-y				
D-d				
Open-delta				

(II) What are the applications of each connection?

(III) What are the advantages and disadvantages of each connection?

Faculty of Engineering

Electrical Power and Machines Engineering Dept.

Mid-Term Exam – First Semester 2015-2016

Course: EPM3111(Electrical Machines 2)	**Time allowed:** 60 minutes
Year: 3rd Electrical Power and Machines Eng.	**Date:** 28/11/2015

Attempt to solve the following questions

Remarks: *Answers should be supported by sketches as you can*

Question 1 — **10 Points (4+6)**

I. What are the properties of the ideal transformer?

II. A 22-kVA, 2200/1100-V, step-down ideal transformer delivers a rated load at a lagging power factor of 0.8. Determine:

a) the secondary winding current,
b) the primary winding current,
c) the impedance on the secondary side as a parallel combination of R_p and X_p
d) the impedance on the primary side as a series combination of R_s and X_s

Question 2 — **15 Points (4+4+7)**

I. Sketch the variation of:

a) the secondary voltage of a transformer against the secondary current at different load power factor
b) the efficiency of a transformer against the load current at different load power factor

II. What are transformer taps? Why are they used? Classify the types of transformers according to the mechanism of tap changing.

III. A 100-kVA, 11,000/220-V distribution transformer has four 2.5% taps on its primary winding.

a) What are the voltage ratios of this transformer at each tap setting?
b) Draw the tap arrangement of the transformer.

Question 3 — **25 Points (16+5+4)**

I. A 100-kVA 2500/125 V, 50Hz, step-down xfr has the following parameters:

$R_H = 1.5\ \Omega$ $X_H = 2.8\ \Omega$

$R_L = 0.015\ \Omega$ $X_L = 0.02\ \Omega$

$R_{CH} = 5000\ \Omega$ $X_{MH} = 3000\ \Omega$

The transformer delivers 85% of the rated load at a terminal voltage of 110V and a power factor of 0.866 lagging. Determine:

a) the core loss and the copper loss
b) the efficiency and the voltage regulation at this load conditions
c) the per-unit loading at maximum efficiency and what is the value of maximum efficiency of the transformer
d) the worst case voltage regulation and the power factor at this condition

II. What is an autotransformer? List its advantages and drawbacks.

III. Draw sketches showing how the transformer given in Question 3-I can be connected as a step-up or step-down autotransformer.

Tanta University Faculty of Engineering Course: Electric Machines (2) Code: EPM3111	Quiz (Model-B)	Date: 24-11-2015 Time allowed: 20 minutes Total score: 10 Dr. *M. El-Nemr* & Dr. *S. Dabour*

Student Name .... Section

Fill the table with the correct answer/answers for the following statements

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answers																				

[1] **No load losses in a transformer can be minimized by using steel of**
a) high silicon content and very thin lamination
b) only high silicon content
c) only very thin lamination
d) low silicon content and very thin lamination

[2] **The tap changer in an electrical power transformer is provided on**
a) both LV and HV winding
b) LV winding
c) either LV or HV winding
d) HV winding

[3] **In a transformer the voltage regulation will be zero when it operates at**
a) unity p.f.
b) leading p.f.
c) lagging p.f.
d) zero p.f. leading.

[4] **A two-winding single phase transformer has a voltage regulation of 4.5% at full-load and unity power-factor. At full-load and 0.80 power-factor lagging load the voltage regulation will be**
a) 4.5%
b) less than 4.5%
c) more than 4.5%
d) 4.5% or more than 4.5%

[5] **A 1:5 step-up transformer has 120 V across the primary and 600 ohms resistance across the secondary. Assuming 100% efficiency, the primary current equals**
a) 0.2 Amp
b) 5 Amps
c) 10 Amps
d) 20 Amps

[6] **In a three phase transformer, if the primary side is connected in star and secondary side is connected in delta, what is the angle difference between phase voltages in the two cases?**
a) delta side lags by -30°
b) star side lags by -30°
c) delta side leads by 30°
d) star side leads by -30°

[7] **The polarity test is not necessary for the single-phase transformer so as to correctly determine ____________ of the transformer.**
a) excitation branch parameters
b) transformation ratio
c) series branch parameters
d) parallel connection

[8] **When two transformers are operating in parallel, they will share the load as under:**
a) proportional to their impedances
b) inversely proportional to their impedances
c) 50% - 50%
d) 25%-75%

[9] **The maximum efficiency for a transformer occurs at 80% of full load. Its core loss is Pc and copper loss is P_{cu} at full load. For this transformer, the ratio of P_{cu}/P_c is**
a) 0.8
b) 1.25
c) 0.64
d) 1.5625

[10] **Why HV winding is always placed above the LV winding in a core type transformer?**
a) To reduce leakage flux
b) To reduce insulation cost
c) To provide better cooling
d) To provide more mechanical strength

[11] **If a transformer primary is connected to a square wave voltage source, its output voltage will be**
a) A square wave
b) A sine wave
c) A triangular wave
d) A pulsed wave

[12] **If in a substation there is one 132 / 33 kV transformer whose secondary is connected with primary of one 33 / 11 kV transformer, the total transformation ratio of the substation will be**
a) 7
b) 12
c) 4
d) 3

[13] **The primary winding of a *220/6 V, 50 Hz* transformer is supplied from *110 V, 60 Hz* source. The secondary output voltage will be**
a) 3.6 V
b) 2.5 V
c) 3.0 V
d) 6.0 V

[14] **The emf induced in the primary of a transformer**
a) is in phase with the flux
b) lags behind the flux by 90°
c) leads the flux by 90°
d) is in phase opposition to that of flux

[15] **A 5 KVA, 220/440 V, *50 Hz*, single phase transformer connected to 220V, *40 Hz* supply with secondary winding open circuited. Then**
a) Both eddy current and hysteresis losses decreases.
b) Both eddy current and hysteresis losses increases.
c) Eddy current loss remains the same, but hysteresis loss increases.
d) Eddy current loss increases, but hysteresis loss remains the same.

[16] **A 800 kVA, single phase transformer has a voltage ratio of 6600 / 5000 V. If the emf per turn is 8 V, calculate the number of turns on high voltage and low voltage side.**
a) 825 & 625
b) 625 & 825
c) 600 & 800
d) 500 & 660

[17] **Secondary current of a step down transformer is**
a) lower than primary current
b) higher than primary current
c) equal to primary current
d) double than primary current

[18] **The open delta connection of the three-phase transformer is capable of delivering a power reduced by ________ without overloading the transformer.**
a) 58 %
b) 42 %
c) 50 %
d) 80 %

[19] **Full load copper loss in transformer is 1600 W. At half load the loss will be**
a) 400 W
b) 1600 W
c) 3200 W
d) 6400 W

[20] **Distribution transformers are usually designed to have maximum efficiency at**
a) 50 % of full load
b) 85 % of full load
c) near full load
d) near no load

Tanta University
Faculty of Engineering
Course: Electric Machines (2)
Code: EPM3111

Quiz (Model - B)

20

Date: 8 -11-2016
Time allowed: 20 minutes
Total score: 20
Dr. Sherif M. Dabour

Student Name .... Section ....

Q1) Fill the table with the correct answer/answers for the following statements

Question	1	2	3	4	5	6	7	8	9	10
Answers										

[21] In the parallel opearation of two transformers, they should have the same
e) kVA ratings
f) voltage ratio
g) per unit impedance
h) polarity.

[22] If the impedance triangles of two transformers operating in parallel are not identical in shape and size, the two transformers will
a) share the load unequally
b) get heated unequally
c) have a circulatory secondary current
d) run with different power factors.

[23] A 1:5 step-up transformer has 120 V across the primary and 600 ohms resistance across the secondary. Assuming 100% efficiency, the primary current equals
e) 0.2 Amp
f) 5 Amps
g) 10 Amps
h) 20 Amps

[24] When two transformers are operating in parallel, they will share the load as under:
e) proportional to their impedances
f) inversely proportional to their impedances
g) 50% - 50%
h) 25%-75%

[25] A transformer transforms
e) voltage
f) impedance
g) frequency
h) current

[26] Transformer core is laminated in order to
a) simplify its construction
b) minimize eddy current loss
c) reduce cost
d) reduce hysteresis loss

[27] The primary winding of a 2200/120 V, 50 Hz transformer is supplied from 1100 V, 60 Hz source. The secondary output voltage will be
e) 60.0 V
f) 50 V
g) 72.0 V
h) None of these

[28] In a transformer, the leakage flux of each winding is proportional to the current in that winding because
a) Ohm's law applies to magnetic circuits
b) leakage paths do not saturate
c) the two windings are electrically isolated
d) mutual flux is confined to the core.

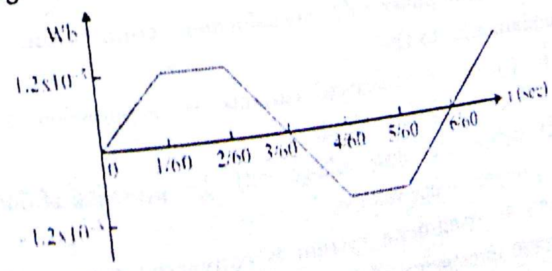
[29] A 5 KVA, 220/440 V, 50 Hz, single phase transformer connected to 220V, 40 Hz supply with secondary winding open circuited. Then
e) Both eddy current and hysteresis losses decreases.
f) Both eddy current and hysteresis losses increases.
g) Eddy current loss remains the same, but hysteresis loss increases.
h) Eddy current loss increases, but hysteresis loss remains the same.

[30] A 800 kVA, single phase transformer has a voltage ratio of 6600 / 5000 V. If the emf per turn is 8 V, calculate the number of turns on high voltage and low voltage side.
e) 825 & 625
f) 625 & 825
g) 600 & 800
h) 500 & 660

Q2) State true (√) or false (×) and correct the false statements

Statement	Ans.	Remarks
(1) If the rating of the transformer is 60-Hz, the transformer may be operated at 50-Hz with a 60/50 percent higher voltages if this action does not cause insulation problems.		
(2) If the secondary winding of the ideal transformer has N_p turns, the number of turns in the primary winding for maximum power transfer to the load will be $0.5N_p$ turns when the supply and load has impedances of R and 4R respectively.		
(3) The transformer draw a no-load current when its secondary is open.		
(4) In an ideal transformer, the terminal voltages are in-phase.		
(5) The currents in the windings of an ideal transformer are inversely proportional to the turns of the windings		

Q3) The flux in the core of an ideal 1 ϕ transformer varies with time as shown in Figure. The primary coil has 100 turns and the secondary coil has 400 turns. Sketch the waveform of the induced voltage e_1 in the primary winding.



Tanta University-Faculty of Engineering

Course: EPM3111 Electric Machines (2)

Quiz - 2

Model -A

Date: 13-12-2010

Time allowed: 20 minutes

Student Name --

Section -----

10

Fill the table with the correct <u>answer/answers</u> for the following statements

Question	1	2	3	4	5	6	7	8	9	10
Answers										

[31] The marked increase in kVA capacity produced by connecting a 2-winding transformer as an autotransfomer is due to

i) increase in turn ratio
j) increase in secondary voltage
k) increase in transformer efficiency
l) conductive link between sides.

[32] The saving in Cu achieved by converting a 2-winding transformer into an autotransformer is determined by

e) voltage transformation ratio
f) load on the secondary
g) magnetic quality of core material
h) size of the transformer core.

[33] A 480/120 V, 5 kVA, two-winding transformer is to be used as an autotransformer to supply power at 480 V from 1,600 V supply. The kVA rating of the autotransformer will be

i) 5 j) 15 k) 25 l) None of these.

[34] An autotransformer having a transformation ratio of 0.8 supplies a load of 3 kW. The power transferred conductively from primary to secondary is .... kW.

a) 0.6 b) 2.4 c) 1.5

[35] An autotransformer having a transformation ratio of 0.8 supplies a load of 10 kW. The kW transferred inductively from primary to the secondary is

a) 10 b) 8 c) 2 d) None of these.

[36] In a three-phase Yy transformer connection, neutral is fundamental to the

i) passage of unbalanced currents due to unbalanced loads
j) suppression of harmonics
k) balancing of phase voltages with respect to line voltages
l) provision of dual service.

[37] When an open-delta system is converted into a Dd system, increase in capacity of the system is .... percent.

a) 57.7 b) 86.6 c) 42.3 d) 73.3

[38] When a Dd bank is converted into an open-delta connection, each of the two remaining transformers supplies .... percent of the original load.

a) 57.7 b) 28.85 c) 42.3 d) 73.3

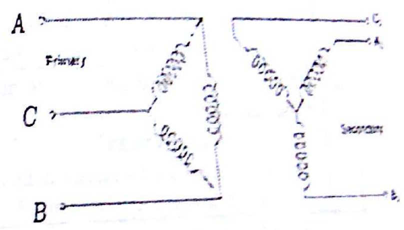
[39] In a three-phase Dy transformer shown in Fig., the phase displacement of secondary line voltages with corresponding primary line voltages will be

a) Zero b) 30° lag c) 30° lead d) None of these.

[40] The phase shift of $Yd1$ transformer is ....

a) Zero
b) -30°
c) 30°
d) None of these.

Best wishes - Dr. Sherif Dabour



Tanta University
Faculty of Engineering
Electrical Power and Machines
Engineering Dept.

Mid-Term Exam – First Semester 2016-2017

Course: EPM3111(Electrical Machines 2)	**Time allowed: 60 minutes**
Year: 3rd Electrical Power and Machines Eng.	**Date: 12/11/2016**
	Total Score: 50

Question 1 — **25 Points (10+5+5+5)**

I "The transformer draws some current when its secondary winding is open."

c) Why does the transformer draw this current?
d) What are the components of this current?
e) Distinguish between the components of this current.
f) How these components are modeled in the transformer equivalent circuit?

II "Some of the flux created by the transformer windings known as the leakage flux, leaves the core and complete its path through the air."

a) How can the leakage flux be minimized?
b) Is it possible to have no leakage flux?
c) Why is it modeled in the equivalent circuit as "series" "inductor"?

III "In a distribution transformer, the maximum efficiency occurs at 50% of the full-load".

a) What does this mean?
b) If the operating frequency is increased, what happens to the load current at maximum efficiency?

IV When a transformer is connected to a 1000-V, 50-Hz supply the core loss is 1000 W, of which 650 is hysteresis and 350 is eddy current loss. If the applied voltage is raised to 2,000 V and the frequency to 100 Hz, find the new core losses.

Question 2 — **25 Points (5+20)**

I Sketch the variation of:

a) the voltage regulation of a transformer against the secondary current at different load power factor
b) the efficiency of a transformer against the load current at different load power factor

II A 6-kVA 250/500 V, 50Hz, step-up transformer has the following parameters:

$R_{e1} = 0.174\ \Omega$ $\qquad X_{e1} = 0.3786\ \Omega$

$R_C = 781.25\ \Omega$ $\qquad X_M = 314.35\ \Omega$

f) If the standard no-load and short-circuit tests are performed on this transformer. What are the approximate reading of the instruments?
g) Determine the primary voltage, voltage regulation and efficiency at full-load conditions and 0.8 power factor lagging?
h) Find the worst-case voltage regulation of this transformer?
i) Calculate the per-unit loading at maximum efficiency? Also determine the maximum efficiency?
j) At what per-unit loading does the effeciency is 95% at 0.8 pf lagging.
k) Determine the ALL-DAY efficiency. If the transformer has the following load-cycle: No load for 6 hours, 70% full-load for 10 hours at 0.8 power factor and 90% full-load for 8 hours at 0.9 leading power factor.

Wish you all the best **Prof. Ahmed Shobier** and **Dr. Sherif Dabour**

Tanta University Faculty of Engineering Electrical Power and Machines Engineering Dept.	
Final Exam – First Semester 2016-2017	
Course: EPM3111(Electrical Machines 2)	Time allowed: 3 hours
Year: 3rd Electrical Power and Machines Eng.	Date: 12/1/2017
No. of Pages: 4	Total Score: 120

Attempt to solve the following questions.
Answers should be supported by sketches as you can

Question 1	25 Points [1×25]

Tick the correct answer for the following statements: *(Verification of your choice is A MUST when numerical data are given).*

[41] A transformer does not possess __________ changing property.
a) impedance b) voltage c) current

[42] Transformer core is laminated in order to
a) simplify its construction b) minimize eddy current loss
c) reduce cost d) reduce hysteresis loss.

[43] In the transformer _______ winding has got more cross sectional area.
a) high voltage b) low voltage c) primary

[44] If a transformer primary is connected to a square wave voltage source, its output voltage will be
a) square wave b) sine wave c) triangular

[45] At 50Hz operation, a single-phase transformer has hysteresis loss of 200W and eddy current loss of 100W. Its core loss at 60Hz operation will be
a) 432 W b) 406 W c) 384 W

[46] The primary winding of a 2200/120V, 50Hz transformer is supplied from 1100V, 60Hz source. The secondary output voltage will be
a) 60.0 b) 50 c) 72.0 d) None of these.

[47] A 2kVA transformer has iron loss of 150W and full-load cupper loss of 250W. The maximum efficiency of the transformer would occur when the total loss is
a) 500 W b) 400 W c) 300 W d) None of these.

[48] In a transformer, if the iron losses and copper losses are 40.5kW and 50kW, respectively, then at what fraction of load will the efficiency be maximum?
a) 0.60 b) 0.57 c) 0.70

[49] An ideal power transformer will have maximum efficiency at a load such that
a) copper loss is less than iron loss b) copper loss is equal to iron
c) copper loss is higher than iron d) none of the these.

[50] Efficiency of power transformer is of the order of
a) 100% b) 98% c) 50%

[51] Power transformers are designed to have maximum efficiency is
a) 100% b) 98% c) 50%

[52] The efficiency of a transformer at full load 0.8 power factor lagging is 95%. Its efficiency at full load 0.8 pf leading will be
a) 80% b) 90% c) 95%

[53] A 5KVA, 220/440V, 50Hz, single phase transformer connected to 220V, 40Hz supply with secondary winding open circuited. Then
a) Both eddy current and hysteresis losses decreases
b) Both eddy current and hysteresis losses increases
c) Eddy current loss remains the same, but hysteresis loss increases
d) Eddy current loss increases, but hysteresis loss remains the same.

[54] When two transformers are operating in parallel, they will share the load as under:
m) proportional to their impedances n) inversely proportional to their impedances
o) 50% - 50% p) 25%-75%

[55] In parallel operation of two single-phase transformers, if the impedance triangles of the transformers are not identical in shape and size
a) power factors at which the transformers operate will be same but different from the load p.f.
b) power factors at which the transformers operate and the load p.f. will be the same.
c) power factor of one transformer and the power factor of common load will be the same.
d) power factors at which the transformers operate will be different from one

another and again these will be different from the power factor of common load.

[56] In a transformer, the leakage flux of each winding is proportional to the current in that winding because

e) Ohm's law applies to magnetic circuits
f) leakage paths do not saturate
g) the two windings are electrically isolated
h) mutual flux is confined to the core.

[57] In a transformer fed from a fundamental frequency of voltage source, the source of harmonics is the

a) overload
b) poor insulation
c) iron loss

[58] The marked increase in kVA capacity produced by connecting a 2-winding transformer as an autotransformer is due to

m) increase in turn ratio
n) increase in secondary voltage
o) increase in transformer efficiency
p) conductive link between sides.

[59] The saving in Cu achieved by converting a 2-winding transformer into an autotransformer is determined by

i) voltage transformation ratio
j) load on the secondary
k) magnetic quality of core material
l) size of the transformer core.

[60] A 480/120V, 5kVA, two-winding transformer is to be used as an autotransformer to supply power at 480 V from 1,600 V supply. The kVA rating of the autotransformer will be

a) 5
b) 15
c) 25
d) None

[61] An autotransformer having a transformation ratio of 0.8 supplies a load of 3 kW. The power transferred conductively from primary to secondary is .... kW.

e) 0.6
f) 2.4
g) 1.5

[62] An autotransformer having a transformation ratio of 0.8 supplies a load of 10 kW. The kW transferred inductively from primary to the secondary is

e) 10
f) 8
g) 2
h) None

[63] When an open-delta connection is converted into a Dd bank, the power rating is increased by .... of the original load.

e) 57.7%
f) 173.3%
g) 42.3%
h) 73.3%

[64] In a three-phase Yy transformer connection, neutral is fundamental to the

a) passage of unbalanced currents due to unbalanced loads
b) suppression of harmonics
c) balancing of phase voltages with respect to line voltages
d) provision of dual service.

[65] In a three-phase Dy transformer shown in Fig., the phase displacement of secondary line voltages with corresponding primary line voltages will be

c) Zero
f) 30^0 lag
g) 30^0 lead
h) None

Question 2

25 Points [5+4+10+6]

[A] State true (✓) or false (×) and correct the false statements

(6) The transformer draw a no-load current when its secondary is shorted.

(7) In practical transformer, the terminal voltages are in-phase.

(8) The currents in the windings of an ideal transformer are inversely proportional to the turns of the windings

(9) If the rating frequency of the transformer is 50-Hz, the transformer may be operated at 60-Hz with a 60/50 percent higher voltages if this action does not cause insulation problems.

(10) If the secondary winding of the ideal transformer has N_p turns, the number of turns in the primary winding for maximum power transfer to the load will be $0.5N_p$ turns when the supply and load has impedances of 4R and R respectively.

[B] What differentiates a core-type transformer from a shell-type transformer? In both types, the primary and secondary windings are wrapped one on top of the other, what are the purposes of this approach? (support your answer with suitable sketches)

[C] What happens to a transformer when it is first connected to a power line? Can anything be done to mitigate this problem?

[D] Complete the following table with either sinusoidal (S) or distorted (D):

Three-phase transformer connection	Line voltage	Phase voltage	Line current	Phase current
Yy				
Dy				
Yyn				

Question 3

30 Points [3×10]

A 120 kVA, 2400/240 V, step-down transformer has the following parameters: R_1= 0.75 ohm, X_1 = 0.8 ohm, R_2 = 0.01 ohm, X_2 = 0.02 ohm. The transformer is designed to operate at maximum efficiency at 70% of its rated load with unity pf. Determine:

1) the kVA of the transformer at maximum efficiency,
2) the efficiency at full-load and 0.8 pf lagging,
3) the equivalent core-loss resistance,
4) if the standard no-load and short-circuit tests are performed on this transformer. What are the approximate reading of the instruments?
5) if the transformer is connected to a load whose power-factor varies. Determine the worst-case voltage-regulation at *full-load conditions* and draw the corresponding phasor diagram.
6) find the transformer voltage-regulation at 50% of full-load condition and 0.8 power factor lag and draw the corresponding phasor-diagram.
7) if the transformer has the following load-cycle:

   No load for 6 hours

   70% full load for 10 hours at 0.8 lagging power factor

   90% full load for 8 hours at 0.9 leading power factor

   Determine the ALL-DAY efficiency of the transformer.
8) taking the transformer rating as a base, derive the approximate equivalent circuit of the transformer in per-unit.
9) recalculate part (2) using the per-unit values.
10) draw a sketch showing how this transformer can be reconnected as a 2400/2640V autotransformer. Also, determine (I) the power rating S_{IO}, (II) the power transferred by conduction, (III) the power transferred by induction, and (IV) the saving in copper weight

Question 4	20 Points

A three-phase transformer is assembled by connecting three 720-VA, 360/120-V, single-phase transformers. The constants for each transformer are R_1 = 18.9 Ω, X_1 = 21.6 Ω, R_2 = 2.1 Ω, X_2 = 2.4 Ω, R_c = 8.64 kΩ, and X_m = 6.84 kΩ.

For each of the following configurations a) Yy and b) Yd:

a) Determine the nominal voltage and power ratings of the three-phase transformer. (Please list your answer in a table)

b) Draw the winding arrangements and the per-phase equivalent circuit for each configuration.

Question 5	20 Points [10+(4+4+2)]

a) Derive an expression for the leakage inductance of a transformer L.V. winding of height a_1, width b_1, number of turns N_1, mean length of turn L_{mt} and gap c_1 between L.V. and H.V. windings.

b) Assume the following data for a 250 kVA, 6600/415V, 50 Hz, 3-phase star connected transformer: approximate voltage per turn =9V, maximum flux density=1.25T, cruciform shaped iron core, stacking factor=0.9, window space factor=0.27, window height is double window width, current density= 2.5 million A/m². Determine:

i. circumscribing circle diameter

ii. window height

iii. from the ratio between volt per turn and kVA or otherwise, find the ratio of magnetic loading to electric loading.

Wish you all the best   Prof. Ahmed Shobier & Dr. Sherif Dabour